arXiv:1012.5310v2 [hep-ph] 15 Nov 2011

CERN-PH-TH/2010-322
CPHT-RR 112.1210

Beyond the MSSM Higgs with d=6 effective operators.

I. Antoniadis[a,b], **E. Dudas**[b,c], **D. M. Ghilencea**[a,b,d], **P. Tziveloglou**[b,1]

[a]Department of Physics, CERN - Theory Division, 1211 Geneva 23, Switzerland.

[b]Centre de Physique Théorique, Ecole Polytechnique, CNRS, 91128 Palaiseau, France.

[c]LPT, UMR du CNRS 8627, Bât 210, Université de Paris-Sud, 91405 Orsay Cedex, France.

[d]DFT, National Institute of Physics and Nuclear Engineering (IFIN-HH) Bucharest MG-6, Romania.

Abstract

We continue a previous study of the MSSM Higgs Lagrangian extended by all effective operators of dimension $d = 6$ that can be present beyond the MSSM, consistent with its symmetries. By supersymmetry, such operators also extend the neutralino and chargino sectors, and the corresponding component fields Lagrangian is computed onshell. The corrections to the neutralino and chargino masses, due to these operators, are computed analytically in function of the MSSM corresponding values. For individual operators, the corrections are small, of few GeV for the constrained MSSM (CMSSM) viable parameter space. We investigate the correction to the lightest Higgs mass, which receives, from *individual* operators, a *supersymmetric* correction of up to 4 (6) GeV above the 2-loop leading-log CMSSM value, from those CMSSM phase space points with: EW fine tuning $\Delta < 200$, consistent with WMAP relic density (3σ), and for a scale of the operators of $M = 10\,(8)$ TeV, respectively. Applied to the CMSSM point of minimal fine tuning ($\Delta = 18$), such increase gives an upper limit $m_h = 120(122) \pm 2$ GeV, respectively. The increase of m_h from individual operators can be larger ($\sim 10 - 30$ GeV) for those CMSSM phase space points with $\Delta > 200$; these can now be phenomenologically viable, with reduced Δ, and this includes those points that would have otherwise violated the LEP2 bound by this value. The neutralino/chargino Lagrangian extended by the effective operators can be used in studies of dark matter relic density within extensions of the MSSM, by implementing it in public codes like micrOMEGAs.

[1] E-mail addresses: Ignatios.Antoniadis@cern.ch, Emilian.Dudas@cpht.polytechnique.fr, Dumitru.Ghilencea@cern.ch, pantelis.tziveloglou@gmail.com

Contents

1 Introduction

The physics of the Higgs sector plays a central role in the Standard Model (SM) and its minimal supersymmetric version (MSSM). Its discovery would clarify the (electroweak (EW)) gauge symmetry breaking mechanism. In the supersymmetric case it could also provide an insight into the dark matter problem, due to its link with the MSSM neutralino sector (higgsinos/gauginos), whose LSP is a dark matter candidate. Higgs physics can then be related to both small and large scale physics for which EW and dark matter constraints can be relevant.

The Higgs sector of the MSSM is the minimal that can be constructed in a supersymmetric context. Its EW vev triggered by radiative EW symmetry breaking is strongly related by quantum corrections to the scale of supersymmetry breaking and the mass of superpartners. No discovery of light superpartners will indicate a fine tuning [1, 2] of the EW scale in the MSSM to levels phenomenologically unacceptable and will question supersymmetry as a solution to the hierarchy problem. In constrained MSSM (CMSSM) at 2-loop leading-log (LL), a Higgs of mass of 120 GeV would mean an EW fine tuning $\Delta = 100$ (i.e. 1 part in 100) [3]. Due to quantum corrections (largely QCD ones), Δ grows exponentially, so for $m_h = 126$ GeV, the fine tuning worsens and becomes $\Delta = 1000$. Interestingly enough, a minimization

of Δ at 2-loop, with all theoretical and experimental constraints, except the LEP2 bound [4] on m_h and the WMAP result [5], predicts a value for m_h just above the LEP2 bound, $m_h = 114 \pm 2$ GeV with an acceptable fine tuning, $\Delta = 9$ [3]. This is only mildly changed when one imposes a saturation of dark matter relic density within 3σ, to $m_h = 115.9 \pm 2$ GeV, for a fine tuning $\Delta = 18$, still an acceptable value. The question remains though if such results for m_h are stable under corrections from new physics that may be missed by the otherwise minimal construction of the MSSM higgs sector. It is also interesting to investigate what happens if m_h is not found experimentally near the value predicted by minimal Δ shown above. Can one still have a low fine tuning for m_h above these values? In other words, a large amount of fine tuning that we mentioned for $m_h > 120$ GeV may be taken to indicate that the Higgs sector Lagrangian is not complete, and that new physics beyond this sector can exist, so that its effects could reduce Δ to acceptable values even for $m_h > 120$ GeV (for an example see [6]). If possible, such new physics can be described, in a model independent way, by higher dimensional operators. These operators respect all the symmetries of the MSSM. For practical purposes one can consider operators of dimensions $d = 5$ and $d = 6$, and this paper is a continuation of the work in this direction, started in [7]. For studies of effective operators of $d = 6$ in the MSSM Higgs sector see [8, 7] and [9]-[23] for studies of effective operators in a related context.

There is only one gauge invariant effective supersymmetric operator of $d = 5$ beyond the MSSM Higgs sector that can depend on Higgs and gauge fields only, but the number of similar $d = 6$ operators, is much larger and their analysis is difficult. Ignoring the $d = 5$ operator, the $d = 6$ operators could indicate that the MSSM Higgs, and, by supersymmetry, neutralino sector, are stable under "new physics" corrections, since these are strongly suppressed, by $\sim 1/M^2$ (M is the scale of new physics). One would like to clarify if this is true. However, the extra scale suppression of $d = 6$ operators (relative to $d = 5$ ones) can be compensated by a large $\tan\beta$, and then the $d = 5$ and $d = 6$ operators can have comparable effects. There are stronger motivations to consider $d = 6$ operators. New physics beyond MSSM Higgs and neutralino sectors can arise, in the leading order, as a $d = 6$ operator, without any $d = 5$ one. For example, integrating a massive $U(1)'$ gauge boson generates a $d\!=\!6$ operator in the leading order, but no $d = 5$ one. The convergence of the expansion in $1/M$ is another motivation for studying both $d\!=\!5$ and $d\!=\!6$ operators, if they are generated by the same physics.

For the effective operators expansion to work the scale of new physics should be high enough, to avoid current experimental constraints. Usually EW constraints (ρ parameter) indicate a value $M \sim 8$ TeV or larger [14]. The expansion parameter $\tilde{m}/M$, for $d\!=\!5$ case, and $(\tilde{m}/M)^2$ for $d\!=\!6$ case, where $\tilde{m}$ is any low scale of the model (EW vev, μ parameter, m_0: Susy breaking scale, $m_{1,2}$ gaugino masses) should be less than unity. If this is not true, the effective approach is unreliable, and unintegrated states (that generated the effective operators) should be used instead.

The coefficients of the effective operators can also be constrained, in a global fit of MSSM plus effective operators, by dark matter experiments, due to their implications for the neutralino sector whose LSP is a dark matter candidate. This can be translated in constraints on the corrections to the Higgs mass. Therefore, the overlap of complementary EW and dark matter constraints on new physics would be welcome for model building. As a first step in this direction, in this work we compute in component fields, for the first time, the most general extension of the MSSM Lagrangian in the neutralino and chargino sectors extended by all allowed $d = 5$ and $d = 6$ effective operators. This is related, by supersymmetry, to the corresponding MSSM Higgs Lagrangian with such effective operators, computed in [7].

We then calculate (analytically) the corrections to the neutralino and chargino masses, in the leading order, $1/M^2$, in function of the MSSM corresponding values. It turns out that the supersymmetric mass corrections to the LSP from *individual* operators of $d = 6$ are in general small, of few GeV and less than $1-2\%$ for a scale $M = 5$ to 8 TeV, and as a result, the same is true about the change of the LSP composition relative to the MSSM case. This can change in cases when all operators of a given order are present simultaneously, and then their combined effect is enhanced. To avoid ambiguities we keep the coefficients of all effective operators as independent, so that one can turn on/off some of them, depending on the details of the model considered. The neutralino Lagrangian extended by effective operators is useful in studying the dark matter relic density in MSSM extensions, by implementing it in micrOMEGAs [24].

We also perform a careful investigation of the size of corrections to the mass m_h of the lightest MSSM Higgs field, due to effective operators. In [7] analytical formulae for these corrections were obtained, in $1/M^2$ order, followed by a simple numerical estimate in a very special case and under simplifying assumptions. Here we improve this numerical study by performing a general and accurate numerical analysis of the corrections to m_h, analyzed separately for *individual* operators and including *quantum corrections*, not considered before. We do so by considering the CMSSM phase space points that respect current theoretical and experimental constraints, both electroweak and dark matter ones (except the LEP2 bound on m_h that is not imposed), and treat the effective operators corrections as a perturbation on this "background". We show that the CMSSM points with smallest fine tuning ($\Delta < 200$) are rather stable under (Susy) corrections from the effective operators. The correction to the 2-loop leading-log CMSSM Higgs mass m_h, due to individual operators of $d = 6$, is found to be in the region of up to: 4 GeV (6 GeV) for a scale of new physics near 10 TeV (8 TeV), respectively. With the above remarks on neutralino sector, we could expect that their dark matter relic density is unlikely to be affected, but a more careful analysis is needed for this. Regarding CMSSM phase space points with large EW fine tuning $\Delta > 200$, they give a larger increase ($\sim 10-30$ GeV) of m_h, especially for those m_h otherwise under the LEP2 bound (i.e. ruled out in CMSSM), so that the corrected value of m_h can be brought above this bound. For some but not all operators, this value is still close to 120 GeV. Therefore points ruled out

in the CMSSM by the LEP2 bound or by large EW Δ, can become viable phenomenologically and their EW fine tuning will be reduced, once m_h received a significant classical correction.

The plan of the paper is as follows. Section 2 and 3 compute the onshell total Lagrangian, of the MSSM Higgs, neutralino and chargino sectors, extended by effective operators. Section 4 presents the mass corrections to these fields, with phenomenological results given in Section 5.

2 The Lagrangian of the model.

To begin with, consider the MSSM Higgs sector Lagrangian plus all independent operators of dimensions $d = 5$ and $d = 6$ that are allowed in this sector by the MSSM symmetries. In this section we compute this extended Lagrangian in component fields, in $1/M^2$ order. Such effective operators parametrize in a model independent way whatever new physics may exist in this sector, above $M \sim$ few TeV. All operators are considered here with independent coefficients. The Lagrangian is then

$$\mathcal{L} = \int d^2\theta \sum_{i=1,2} z_i(S, S^\dagger)\, H_i^\dagger e^{V_i} H_i + \Big\{ \int d^2\theta\, \mu(1 + B_0 m_0 \theta\theta)\; H_1.H_2 + h.c. \Big\} + \mathcal{K}_0 + \sum_{j=1}^{8} \mathcal{O}_j \quad (1)$$

where $z_i(S, S^\dagger) = 1 - c_i S S^\dagger$, $S = m_0\theta\theta$, $i = 1, 2$ account for Susy breaking in the MSSM Higgs sector, m_0 is the Susy breaking scale, given by $m_0 = \langle F_{hidden} \rangle / M_{Planck}$. $\mathcal{K}_0$ is the only dimension-five operator present up to non-linear field redefinitions [9], while $\mathcal{O}_i$ are $d{=}6$ operators. Further:

$$\begin{aligned}
\mathcal{K}_0 &= \frac{1}{M} \int d^2\theta\; \zeta(S)\, (H_2.H_1)^2 + h.c. \qquad (2)\\
&= \zeta_{10} \left[2\,(h_2.h_1)(h_2.F_1 + F_2.h_1 - \psi_2.\psi_1) - (h_2.\psi_1 + \psi_2.h_1)^2 \right] + \zeta_{11}\, m_0\, (h_2.h_1)^2 + h.c.
\end{aligned}$$

where $H_i \equiv (h_i, \psi_i, F_i)$, $h_1.h_2 = h_1^0 h_2^0 - h_1^- h_2^+$, $(1/M)\,\zeta(S) \equiv \zeta_{10} + \zeta_{11}\, m_0\theta\theta$, so $\zeta_{10}, \zeta_{11} \sim 1/M$. For the conventions used see Appendix A. The list of $d = 6$ operators is [7, 8] (also [11])

$$\begin{aligned}
\mathcal{O}_j &= \frac{1}{M^2} \int d^4\theta\; \mathcal{Z}_j(S, S^\dagger)\; (H_j^\dagger\, e^{V_j}\, H_j)^2, \quad j \equiv 1, 2.\\
\mathcal{O}_3 &= \frac{1}{M^2} \int d^4\theta\; \mathcal{Z}_3(S, S^\dagger)\; (H_1^\dagger\, e^{V_1}\, H_1)\, (H_2^\dagger\, e^{V_2}\, H_2),\\
\mathcal{O}_4 &= \frac{1}{M^2} \int d^4\theta\; \mathcal{Z}_4(S, S^\dagger)\; (H_2.\, H_1)\, (H_2.\, H_1)^\dagger,\\
\mathcal{O}_5 &= \frac{1}{M^2} \int d^4\theta\; \mathcal{Z}_5(S, S^\dagger)\; (H_1^\dagger\, e^{V_1}\, H_1)\; H_2.\, H_1 + h.c.\\
\mathcal{O}_6 &= \frac{1}{M^2} \int d^4\theta\; \mathcal{Z}_6(S, S^\dagger)\; (H_2^\dagger\, e^{V_2}\, H_2)\; H_2.\, H_1 + h.c.\\
\mathcal{O}_7 &= \frac{1}{M^2} \sum_{s=w,y} \frac{1}{16 g_s^2 \kappa} \int d^2\theta\; \mathcal{Z}_7(S, 0)\; \mathrm{Tr}\, (W^\alpha\, W_\alpha)_s\, (H_2.H_1) + h.c.\\
\mathcal{O}_8 &= \frac{1}{M^2} \int d^4\theta\; \Big[\mathcal{Z}_8(S, S^\dagger)\; (H_2\, H_1)^2 + h.c. \Big] \qquad (3)
\end{aligned}$$

where $W^\alpha = (-1/4)\,\overline{D}^2 e^{-V} D^\alpha e^V$ is the chiral field strength of $SU(2)_L$ or $U(1)_Y$ vector superfields V_w and V_y respectively. Also $V_{1,2} = V_w^a(\sigma^a/2) + (\mp 1/2)\,V_y$ with the upper (minus) sign for V_1. The expressions of these operators in component form are given in Appendix A. The coefficients $\mathcal{Z}$ are given by

$$(1/M^2)\ \mathcal{Z}_i(S,S^\dagger) = \alpha_{i0} + \alpha_{i1}\, m_0\, \theta\theta + \alpha_{i1}^*\, m_0\, \overline{\theta\theta} + \alpha_{i2}\, m_0^2\, \theta\theta\overline{\theta\theta}, \quad \text{where} \quad \alpha_{ij} \sim 1/M^2 \tag{4}$$

with α_{ij} numerical coefficients, assumed independent; α_{j0}, α_{j2} with $j = 1,2,3,4$ are real.

The above equations show only the operators polynomial in fields. There are also derivative operators [7] which can be eliminated in the low energy effective theory limit, via general non-linear field redefinitions or via equations of motion [7, 9, 10]. For details how to eliminate these operators see [9, 10]. To give only two examples of such operators, that can be eliminated, consider the D-term $(H_1^\dagger e^V \overline{D}^2 e^{-V} D^2 e^V H_1) \sim (H_1^\dagger \Box H_1)$ and the F-term $\mathrm{Tr}\,(e^V\, W^\alpha\, e^{-V} D^2 (e^V W_\alpha e^{-V})) \sim \mathrm{Tr}\,(W^\alpha \Box W_\alpha)$ where W is the supersymmetric field strength[2]. Such operators can be eliminated up to redefinition of the soft masses, wavefunction renormalization and μ-term redefinition.

After eliminating the auxiliary fields in $\mathcal{L}$, one finds the onshell Lagrangian, which is

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_F + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_{SSB} \tag{5}$$

Eliminating the D-dependent terms in $\mathcal{L}$ one finds, with the notations in Appendix A, see eqs.(A-15) to (A-17), and vector superfields notation $V_s = (\lambda_s, V_{s\,\mu}, D_s^a/2)$, $s = y, w$:

$$\begin{aligned}
\mathcal{L}_D &= \sum_{s=y,w} -\,(1/2)\ D_s^a D_s^a \left[1 + 1/2\,(\alpha_{70}^s\, h_2.h_1 + h.c.)\right] = \Big\{ -\frac{g_2^2}{8}\,(|h_1|^2 - |h_2|^2) \\
&\times \ \Big[\big(1 + 2\tilde\rho_{1,w} + \frac{1}{2}(\alpha_{70}^w h_2.h_1 + h.c.)\big)\,|h_1|^2 - \big(1 + 2\tilde\rho_{2,w} + (1/2)(\alpha_{70}^w\, h_2.h_1 + h.c.)\big)|h_2|^2\Big] \\
&- \ (g_2 \to g_1; \alpha_{70}^w \to \alpha_{70}^y; \tilde\rho_{j,w} \to \tilde\rho_{j,y})\Big\} - \frac{g_2^2}{2}\,\left[1 + \tilde\rho_{1,w} + \tilde\rho_{2,w} + (1/2)(\alpha_{70}^w\, h_2.h_1 + h.c.)\right] |h_1^\dagger h_2|^2 \\
&+ \ g_2/(2\sqrt{2})\,\left[h_1^\dagger T^a h_1 + h_2^\dagger T^a h_2\right]\left[\alpha_{70}^w\,(h_2.\psi_1 + \psi_2.h_1)\,\lambda_w^a + h.c.\right] \\
&+ \ g_1/(2\sqrt{2})\,\Big(h_1^\dagger \frac{-1}{2} h_1 + h_2^\dagger \frac{1}{2} h_2\Big)\left[\alpha_{70}^y\,(h_2.\psi_1 + \psi_2.h_1)\,\lambda_y + h.c.\right]
\end{aligned} \tag{6}$$

Here $h_1.\psi_2 = -\psi_2.h_1 = h_1^0\psi_2^0 - h_1^-\psi_2^+$, $|h_1|^2 = h_1^{0*}h_1^0 + h_1^{-*}h_1^-$, $h_1^\dagger h_2 = h_1^{0*}h_2^+ + h_1^{-*}h_2^0$, etc.

Eliminating the F-dependent terms in $\mathcal{L}$ gives $\mathcal{L}_F$ below (using notation (A-13), (A-14))

$$\begin{aligned}
\mathcal{L}_F &= \mathcal{L}_{F,1} + \mathcal{L}_{F,2} \\
-\mathcal{L}_{F,1} &\equiv |F_1|^2 + |F_2|^2 = |\mu + 2\,\zeta_{10}\, h_1.h_2|^2\ \big(|h_1|^2 + |h_2|^2\big) \\
&+ \Big[\mu\,\Big(|h_1|^2\,\rho_{21} + |h_2|^2\,\rho_{11} + (h_1.h_2)^\dagger\,(\rho_{22} + \rho_{12}) + (\psi_1.h_2)^\dagger\,\rho_{13} + (h_1.\psi_2)^\dagger\,\rho_{23}\Big) + h.c.\Big]
\end{aligned}$$

[2] These operators are often generated as one-loop counterterms even in simplest orbifold compactifications, see [25, 26], after integrating the Kaluza-Klein modes and come multiplied by the compactification volume.

while $\mathcal{L}_{F,2}$ is due to the nontrivial field metric in the Kahler potential:

$$\begin{aligned} -\mathcal{L}_{F,2} &= |\mu|^2\Big[2\left(\alpha_{10}+\alpha_{20}+\alpha_{40}\right)|h_1|^2\,|h_2|^2 + (\alpha_{30}+\alpha_{40})\left(|h_1|^4+|h_2|^4\right) \\ &\quad + 2\left(\alpha_{10}+\alpha_{20}+\alpha_{30}\right)|h_1.h_2|^2 + \left(|h_1|^2+2\,|h_2|^2\right)\left(\alpha_{50}\,h_2.h_1 + h.c.\right) \\ &\quad + \left(2|h_1|^2+|h_2|^2\right)\left(\alpha_{60}\,h_2.h_1+h.c.\right)\Big] \end{aligned} \tag{7}$$

Apart from auxiliary fields contributions, there are also terms which contain space-time derivatives, that contribute to the kinetic terms for Weyl fermions $\psi_{1,2}$, $\lambda^a_{w,y}$ when the neutral singlet Higgses $h^0_{1,2}$, components of $h_{1,2}$, acquire a vev:

$$\begin{aligned} \mathcal{L}_1 &= \alpha_{10}\left[i\,\overline{\psi}_1\overline{\sigma}^\mu\mathcal{D}_\mu\psi_1\,|h_1|^2 + i\,\overline{\psi}_1\overline{\sigma}^\mu\,\psi_1(\,h_1^\dagger\mathcal{D}_\mu h_1) - i\,(h_1^\dagger\psi_1)\,\sigma^\mu\overline{\psi}_1(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_1 + h.c.\right] \\ &+ \alpha_{20}\left[i\,\overline{\psi}_2\overline{\sigma}^\mu\mathcal{D}_\mu\psi_2\,|h_2|^2 + i\,\overline{\psi}_2\overline{\sigma}^\mu\,\psi_2\,(h_2^\dagger\mathcal{D}_\mu h_2) - i\,(h_2^\dagger\psi_2)\,\sigma^\mu\overline{\psi}_2(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_2 + h.c.\right] \\ &+ \alpha_{30}\big[i\,\overline{\psi}_2\overline{\sigma}^\mu\mathcal{D}_\mu\psi_2\,|h_1|^2 + i\,\overline{\psi}_1\overline{\sigma}^\mu\,\psi_1\,(h_2^\dagger\mathcal{D}_\mu h_2) - i\,(h_1^\dagger\psi_1)\,\sigma^\mu\overline{\psi}_2(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_2 + h.c.\big](1/2) \\ &+ \alpha_{30}\big[i\,\overline{\psi}_1\overline{\sigma}^\mu\mathcal{D}_\mu\psi_1\,|h_2|^2 + i\,\overline{\psi}_2\overline{\sigma}^\mu\,\psi_2\,(h_1^\dagger\mathcal{D}_\mu h_1) - i\,(h_2^\dagger\psi_2)\,\sigma^\mu\overline{\psi}_1(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_1 + h.c.\big](1/2) \\ &+ \alpha_{40}\big[i\,(\psi_1.h_2+h_1.\psi_2)\,\sigma^\mu\partial_\mu(\psi_1.h_2+h_1.\psi_2)^\dagger + h.c.\big](1/2) \\ &+ \big\{\alpha^*_{50}\big[i\,h_1^\dagger\,\mathcal{D}_\mu\psi_1\,\sigma^\mu\,(\psi_1.h_2+h_1.\psi_2)^\dagger + i\,(h_2.h_1)^\dagger\,\overline{\psi}_1\overline{\sigma}^\mu\mathcal{D}_\mu\psi_1\big] + h.c.\big\} \\ &+ \big\{\alpha^*_{60}\big[i\,h_2^\dagger\,\mathcal{D}_\mu\psi_2\,\sigma^\mu\,(\psi_1.h_2+h_1.\psi_2)^\dagger + i\,(h_2.h_1)^\dagger\,\overline{\psi}_2\overline{\sigma}^\mu\mathcal{D}_\mu\psi_2\big] + h.c.\big\} \\ &+ \big\{(1/4)\,\alpha^w_{70}\,(h_2.h_1)\big[i\,(\lambda^a_w\sigma^\mu\Delta_\mu\overline{\lambda}^a_w - \Delta_\mu\overline{\lambda}^a_w\overline{\sigma}^\mu\lambda^a_w)\big] + h.c. + (w\rightarrow y)\big\} \end{aligned} \tag{8}$$

When the Higgs fields neutral singlets acquire a vev, these terms bring a wavefunction renormalization of Weyl kinetic terms and a threshold correction to gauge couplings g_2, g_1.

Also, there are terms that contribute to fermions masses, when singlet Higgs fields acquire a vev (we denote $\lambda_{1,2}\equiv g_2\lambda^a_w\,\sigma^a + g_1\,(\mp 1)\lambda_y$, with "-" for λ_1, σ^a: Pauli matrices, $a=1,2,3$):

$$\begin{aligned} \mathcal{L}_2 &= (\alpha_{10}\sqrt{2})\big[-(h_1^\dagger\lambda_1\psi_1)|h_1|^2 - (h_1^\dagger\psi_1)\,h_1^\dagger\lambda_1 h_1\big] - \alpha_{11}m_0(\overline{\psi}_1 h_1)(\overline{\psi}_1 h_1) \\ &+ (\alpha_{20}\sqrt{2})\big[-(h_2^\dagger\lambda_2\psi_2)|h_2|^2 - (h_2^\dagger\psi_2)\,h_2^\dagger\lambda_2 h_2\big] - \alpha_{21}m_0(\overline{\psi}_2 h_2)(\overline{\psi}_2 h_2) \\ &+ (\alpha_{30}/\sqrt{2})\big[-(h_2^\dagger\lambda_2\psi_2)|h_1|^2 - (h_1^\dagger\psi_1)\,h_2^\dagger\lambda_2 h_2 + (1\leftrightarrow 2)\big] - \alpha^*_{31}m_0(h_1^\dagger\psi_1)(h_2^\dagger\psi_2) \\ &+ (\alpha_{50}/\sqrt{2})\big[h_1^\dagger\lambda_1 h_1\,(\psi_1.h_2+h_1.\psi_2) - (h_2.h_1)(h_1^\dagger\lambda_1\psi_1 + \overline{\psi}_1\overline{\lambda}_1 h_1)\big] \\ &+ (\alpha_{60}/\sqrt{2})\big[h_2^\dagger\lambda_2 h_2\,(\psi_1.h_2+h_1.\psi_2) - (h_2.h_1)(h_2^\dagger\lambda_2\psi_2 + \overline{\psi}_2\overline{\lambda}_2 h_2)\big] \\ &- m_0\,(\alpha^*_{51}|h_1|^2 + \alpha^*_{61}\,|h_2|^2)\,\psi_2.\psi_1 - m_0\,(\alpha^*_{51}\,h_1^\dagger\psi_1 + \alpha^*_{61}\,h_2^\dagger\psi_2)(h_2.\psi_1+\psi_2.h_1) \\ &+ (1/4)\alpha^w_{71}m_0(h_2.h_1)(\lambda^a_w\lambda^a_w) + (1/4)\alpha^y_{71}m_0(h_2.h_1)(\lambda_y\lambda_y) + 2\,\alpha^*_{81}\,m_0\,(h_2.h_1)\,(-\psi_2.\psi_1) \\ &+ \alpha_{41}\,m_0\,(h_2.h_1)\,(-\psi_2.\psi_1)^\dagger + \zeta_{10}\big[2\,(h_2.h_1)(-\psi_2.\psi_1) - (h_2.\psi_1+\psi_2.h_1)^2\big] + \text{h.c.} \end{aligned} \tag{9}$$

Further, there are some interaction terms

$$\begin{aligned}\mathcal{L}_3 = & -\alpha_{10}\,(\overline{\psi}_1\psi_1)(\overline{\psi}_1\psi_1) - \alpha_{20}\,(\overline{\psi}_2\psi_2)(\overline{\psi}_2\psi_2) - \alpha_{30}\,(\overline{\psi}_1\psi_1)(\overline{\psi}_2\psi_2) + \alpha_{40}(\psi_2.\psi_1)^\dagger(\psi_2.\psi_1)\\ & + \Big\{\, (1/4)\,\alpha_{70}^w\,\Big[(-1/2)\,(h_2.h_1)\,(F_w^{a\,\mu\nu}F^a_{w\,\mu\nu} + (i/2)\,\epsilon^{\mu\nu\rho\sigma}F^a_{w\,\mu\nu}F^a_{w\,\rho\sigma})\\ & - \sqrt{2}\,(h_2.\psi_1+\psi_2.h_1)\,\sigma^{\mu\nu}\lambda^a_w F^a_{w,\mu\nu} - \psi_2.\psi_1\,\lambda^a_w\lambda^a_w\Big] + (w\rightarrow y) + h.c.\Big\} \end{aligned} \tag{10}$$

with $(\overline{\psi}_1\psi_1)(\overline{\psi_2}\psi_2) = (\overline{\psi_1^0}\psi_1^0 + \overline{\psi_1^-}\psi_1^-)(\overline{\psi_2^0}\psi_2^0 + \overline{\psi_2^+}\psi_2^+)$, etc, where spinor indices are not shown.

Also, there are $h_{1,2}$ dependent terms that contain space-time derivatives, which contribute to the kinetic terms in the Higgs sector, when the singlet Higgs fields acquire a vev:

$$\begin{aligned}\mathcal{L}_4 \;=\;& 2\alpha_{10}\,\big[|h_1|^2\,|\mathcal{D}_\mu h_1|^2 + |h_1^\dagger\mathcal{D}^\mu h_1|^2\big] + 2\alpha_{20}\,\big[|h_2|^2\,|\mathcal{D}_\mu h_2|^2 + |h_2^\dagger\mathcal{D}^\mu h_2|^2\big]\\ +\;& \alpha_{30}\,\big[|h_1|^2\,|\mathcal{D}_\mu h_2|^2 + (h_1^\dagger\mathcal{D}_\mu h_1)(h_2^\dagger\overleftarrow{\mathcal{D}}^\mu h_2) + (1\leftrightarrow 2)\big] + \alpha_{40}\,|\partial_\mu(h_2.h_1)|^2\\ +\;& \big\{\alpha_{50}\big[\,|\mathcal{D}_\mu h_1|^2\,(h_2.h_1) + (h_1^\dagger\overleftarrow{\mathcal{D}}_\mu h_1)\,\partial^\mu(h_2.h_1)\big] + h.c.\big\}\\ +\;& \big\{\alpha_{60}\big[\,|\mathcal{D}_\mu h_2|^2\,(h_2.h_1) + (h_2^\dagger\overleftarrow{\mathcal{D}}_\mu h_2)\,\partial^\mu(h_2.h_1)\big] + h.c.\big\}\end{aligned} \tag{11}$$

Finally, the Lagrangian contains (F and D-independent) corrections due to supersymmetry breaking, i.e. terms proportional to m_0, due to spurion dependence in the higher dimensional operators (of dimensions $d = 5$ and $d = 6$) as well as the usual soft terms of the MSSM. All these together give a final contribution to the Lagrangian:

$$\begin{aligned}\mathcal{L}_{SSB} = -V_{SSB} = \;& m_0^2\,\big[\alpha_{12}\,|h_1|^4 + \alpha_{22}\,|h_2|^4 + \alpha_{32}\,|h_1|^2\,|h_2|^2 + \alpha_{42}\,|h_2.h_1|^2\\ & + \big(\alpha_{52}\,|h_1|^2\,(h_2.h_1) + h.c.\big) + \big(\alpha_{62}\,|h_2|^2\,(h_2.h_1) + h.c.\big)\big]\\ & + \big[\,m_0^2\,\alpha_{82}\,(h_1.h_2)^2 + \zeta_{11}\,m_0\,(h_2.h_1)^2 + \mu\,B_0\,m_0\,(h_1.h_2) + h.c.\big]\\ & - m_0^2\,(c_1|h_1|^2 + c_2|h_2|^2)\end{aligned} \tag{12}$$

This concludes the presentation of the full Lagrangian, in $1/M^2$ order. Additional transformations (fields redefinitions or eqs of motion) can be used to eliminate the non-diagonal kinetic terms of fermions and scalars, to obtain a canonical form.

3 The neutralino and chargino Lagrangian.

From the total Lagrangian of the previous section one can obtain the Lagrangian of the neutralino and chargino sectors. Since the result is long, its detailed form in component fields is provided in Appendix B. In this section we extract from this Lagrangian only the terms that contribute to neutralino masses and their kinetic terms (hereafter $\delta\mathcal{L}_{D,F,2,1}$). These terms

originate from $\mathcal{L}_D$, $\mathcal{L}_{F,1}$, $\mathcal{L}_2$, $\mathcal{L}_1$ of previous section and are present in addition to the MSSM original terms. These are detailed below (in component, gauge-singlet fields notation):

$$\delta\mathcal{L}_D = -\frac{1}{4\sqrt{2}}\big(|h_1^0|^2 - |h_2^0|^2\big)\big(g_2\lambda_w^3\alpha_{70}^w - g_1\lambda_y\alpha_{70}^y\big)\big(h_2^0\psi_1^0 + \psi_2^0 h_1^0\big) + h.c. \tag{13}$$

together with

$$\begin{aligned} -\,\delta\mathcal{L}_{F,1} &= -\frac{\mu}{4}\big(|h_1^0|^2 + |h_2^0|^2\big)\big(\alpha_{70}^w\lambda_w^3\lambda_w^3 + \alpha_{70}^y\,\lambda_y\lambda_y\big) + \mu\psi_1^0\psi_1^0\,(-2\alpha_{10}\,h_1^{0*}h_2^0 + \alpha_{50}h_2^{0\,2}) \\ &+ \psi_1^0\psi_2^0\,\big[-(\mu\alpha_{40} + \mu\,\alpha_{30})(|h_1^0|^2 + |h_2^0|^2) + (\alpha_{50} + \alpha_{60})\,(\mu + \mu)\,h_1^0 h_2^0\,\big] \\ &+ \mu\psi_2^0\psi_2^0(-2\alpha_{20}\,h_1^0 h_2^{0*} + \alpha_{60}h_1^{02}) + h.c. \end{aligned} \tag{14}$$

and

$$\begin{aligned} \delta\mathcal{L}_2 &= (g_2\lambda_w^3 - g_1\lambda_y)(\delta_1\psi_1^0 + \delta_2\psi_2^0) + \delta_3\,\psi_1^0\psi_1^0 + \delta_4\,\psi_1^0\psi_2^0 + \delta_5\,\psi_2^0\psi_2^0 \\ &- \frac{1}{4}\,m_0 h_1^0 h_2^0\,\big(\alpha_{71}^w\lambda_w^3\lambda_w^3 + \alpha_{71}^y\lambda_y\lambda_y\big) + h.c. \end{aligned} \tag{15}$$

where we introduced the notation:

$$\begin{aligned} \delta_1 &= -2\sqrt{2}\alpha_{10}\,|h_1^0|^2\,h_1^{0*} + \sqrt{2}\alpha_{50}\,|h_1^0|^2\,h_2^0 + (\alpha_{50}^*/\sqrt{2})\,h_1^{0\,*\,2}h_2^{0\,*} - (\alpha_{60}/\sqrt{2})\,|h_2^0|^2\,h_2^0 \\ \delta_2 &= 2\sqrt{2}\alpha_{20}\,|h_2^0|^2\,h_2^{0*} - \sqrt{2}\alpha_{60}\,|h_2^0|^2\,h_1^0 - (\alpha_{60}^*/\sqrt{2})\,h_1^{0*}\,h_2^{0*\,2} + (\alpha_{50}/\sqrt{2})\,|h_1^0|^2\,h_1^0 \\ \delta_3 &= -\,\alpha_{11}^*\,m_0 h_1^{0*\,2} + m_0\alpha_{51}^*\,h_1^{0*}h_2^0 - \zeta_{10}\,h_2^{0\,2} \\ \delta_4 &= 2m_0(\alpha_{51}^*|h_1^0|^2 + \alpha_{61}^*|h_2^0|^2) - \alpha_{31}^*\,m_0\,h_1^{0*}h_2^{0*} - 2\alpha_{81}^* m_0 h_1^0\,h_2^0 - \alpha_{41}^* m_0 h_1^{0*}h_2^{0*} - 4\zeta_{10}\,h_1^0\,h_2^0 \\ \delta_5 &= -\,\alpha_{21}^* m_0 h_2^{0*\,2} + m_0\,\alpha_{61}^*\,h_1^0 h_2^{0*} - \zeta_{10}\,h_1^{0\,2} \end{aligned} \tag{16}$$

$\mathcal{L}_1$ and $\mathcal{L}_4$ contain non-canonical, non-diagonal kinetic terms for neutralinos/charginos and neutral higgses, respectively, and these have to be carefully considered. The terms in $\mathcal{L}_1$ that generate non-canonical kinetic terms for neutralinos, are denoted $\delta\mathcal{L}_1$ and are

$$\begin{aligned} \delta\mathcal{L}_1 &= i\,\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0\,\nu_1 + i\,\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_2^0\,\nu_2 + i\,\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu\psi_2^0\,\nu_3 + i\,\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0\,\nu_4 \\ &+ \frac{i}{2}\,\big(\lambda_w^3\sigma^\mu\partial_\mu\overline{\lambda}_w^3 - \partial_\mu\overline{\lambda}_w^3\overline{\sigma}^\mu\lambda_w^3\big)\,\nu_5^w + \frac{i}{2}\,\big(\lambda_y\sigma^\mu\partial_\mu\overline{\lambda}_y - \partial_\mu\overline{\lambda}_y\overline{\sigma}^\mu\lambda_y\big)\,\nu_5^y + h.c. \end{aligned} \tag{17}$$

with

$$\begin{aligned} \nu_1 &= 2\alpha_{10}\,|h_1^0|^2 + (1/2)\,\alpha_{30}\,|h_2^0|^2 + (1/2)\,\alpha_{40}\,|h_2^0|^2 - 2\,h_1^{0*}\,h_2^{0*}\,\alpha_{50}^* \\ \nu_2 &= 2\alpha_{20}\,|h_2^0|^2 + (1/2)\,\alpha_{30}\,|h_1^0|^2 + (1/2)\,\alpha_{40}\,|h_1^0|^2 - 2\,h_1^{0*}\,h_2^{0*}\,\alpha_{60}^* \\ \nu_3 &= (1/2)\,\alpha_{30}\,h_1^0\,h_2^{0*} + (1/2)\,\alpha_{40}\,h_1^0\,h_2^{0*} - \alpha_{60}^*\,h_2^{0\,*\,2} \\ \nu_4 &= (1/2)\,\alpha_{30}\,h_2^0\,h_1^{0*} + (1/2)\,\alpha_{40}\,h_2^0\,h_1^{0*} - \alpha_{50}^*\,h_1^{0\,*\,2} \\ \nu_5^w &= -(1/2)\,\alpha_{70}^w\,h_1^0\,h_2^0, \qquad \nu_5^y = -(1/2)\,\alpha_{70}^y\,h_1^0\,h_2^0 \end{aligned} \tag{18}$$

For our purposes it is useful to re-write this as

$$\begin{aligned}
\delta\mathcal{L}_1 &= \frac{i}{2}\,\nu_{11^*}\,\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0+\frac{i}{2}\,\nu_{22^*}\,\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_2^0+i\,\nu_{43^*}\,\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0\\
&+\frac{i}{2}\,\nu^w_{55^*}\,\lambda^3_w\sigma^\mu\partial_\mu\overline{\lambda}^3_w+\frac{i}{2}\,\nu^y_{55^*}\,\lambda_y\sigma^\mu\partial_\mu\overline{\lambda}_y+\text{h.c.}+S_i^\mu\partial_\mu\nu_i
\end{aligned}\tag{19}$$

S_i^μ is a function of fields, not specified here, and its contribution is vanishing if ν_i is a constant, which is indeed the case when the Higgs fields acquire a vev. Also we introduced:

$$\nu_{ij^*}\;\equiv\;\nu_i+\nu_j^*\tag{20}$$

Adding together (13) to (19) and the original MSSM neutralino/chargino terms ($\delta\mathcal{L}_{mssm}$) we finally have the following part of the neutralino Lagrangian needed for the mass spectrum (that is, without interacting terms, given in Appendix B):

$$\begin{aligned}
\mathcal{L}_\chi &= \delta\mathcal{L}_D+\delta\mathcal{L}_{F,1}+\delta\mathcal{L}_1+\delta\mathcal{L}_2+\delta\mathcal{L}_{mssm}\\
&= \delta\mathcal{L}_D+\delta\mathcal{L}_{F,1}+\delta\mathcal{L}_2\\
&+\Big\{\,\frac{i}{2}\,(1+\tilde\nu_{11^*})\,\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0+\frac{i}{2}\,(1+\tilde\nu_{22^*})\,\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_2^0+i\,\tilde\nu_{43^*}\,\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0\\
&+\frac{i}{2}\,(1+\tilde\nu^w_{55^*})\,\lambda^3_w\sigma^\mu\partial_\mu\overline{\lambda}^3_w+\frac{i}{2}\,(1+\tilde\nu^y_{55^*})\,\lambda_y\sigma^\mu\partial_\mu\overline{\lambda}_y-\frac{1}{\sqrt{2}}(g_2\lambda^3_w-g_1\lambda_y)(h_1^{0*}\psi_1^0-h_2^{0*}\psi_2^0)\\
&-\frac{1}{2}\,m_2\,\lambda^3_w\lambda^3_w-\frac{1}{2}\,m_1\lambda_y\lambda_y-\mu\,\psi_1^0\psi_2^0+\text{h.c.}\,\Big\}
\end{aligned}\tag{21}$$

where $\tilde\nu_{ij^*}$ are the values of ν_{ij^*} when the neutral Higgses acquire a vev:

$$\tilde\nu_{ij^*}\equiv\nu_{ij^*}\Big|_{h_i^0\to v_i/\sqrt{2}}\tag{22}$$

To remove the off-diagonal kinetic terms in $\mathcal{L}_\chi$, we perform a field redefinition[3]:

$$\begin{aligned}
\lambda_y &= (1-\tilde\nu^y_{55^*}/2)\lambda_y^{''},\qquad \lambda^3_w=(1-\tilde\nu^w_{55^*}/2)\lambda_w^{3''}\\
\psi_1^0 &= (1-\tilde\nu_{11^*}/2)\;\psi_1^{0''}-(\tilde\nu_{34^*}/2)\;\psi_2^{0''}\\
\psi_2^0 &= (-\tilde\nu_{43^*}/2)\;\;\psi_1^{0''}+(1-\tilde\nu_{22^*}/2)\psi_2^{0''}
\end{aligned}\tag{23}$$

which only changes the MSSM part of the Lagrangian $\mathcal{L}_\chi$, (ignoring corrections higher than $1/M^2$). In the new basis (double primed) $\mathcal{L}_\chi$ has canonical kinetic terms, but it is not yet

[3]The off-diagonal kinetic terms can also be removed by a unitary transformation followed by a (non-unitary) rescaling of the Weyl fermions. However, the field redefinitions used below provide a simpler form for the final result, as they only affect the MSSM part of L, while the unitary transformation (that turns out to be M-independent) also acts on the rest of the Lagrangian, which results in more complicated final expressions. The two approaches are equivalent, the eigenvalue problem is not changed.

in the form needed to compute the neutralino masses. This is because there also are non-standard Higgs kinetic terms that must be brought to canonical form; adding the MSSM higgs kinetic part, hereafter denoted $\mathcal{L}_{kt}^{MSSM}$, then these terms are [7]

$$\mathcal{L}_4 + \mathcal{L}_{kt}^{MSSM} \supset (\delta_{ij^*} + g_{ij^*})\, \partial_\mu h_i^0\, \partial^\mu h_j^{0*}, \qquad i,j = 1,2. \tag{24}$$

where the field dependent metric is:

$$\begin{aligned} g_{11^*} &= 4\,\alpha_{10}\,|h_1^0|^2 + (\alpha_{30}+\alpha_{40})\,|h_2^0|^2 - 2\,(\alpha_{50}\,h_1^0\,h_2^0 + h.c.) \\ g_{12^*} &= (\alpha_{30}+\alpha_{40})\,h_1^{0*}\,h_2^0 - \alpha_{50}^*\;h_1^{0*2} - \alpha_{60}\;h_2^{0\,2}, \qquad g_{21^*} = g_{12^*}^* \\ g_{22^*} &= 4\,\alpha_{20}\,|h_2^0|^2 + (\alpha_{30}+\alpha_{40})\,|h_1^0|^2 - 2\,(\alpha_{60}\,h_1^0\,h_2^0 + h.c.) \end{aligned} \tag{25}$$

The metric g_{ij^*} is expanded about a background value $\langle h_i^0\rangle = v_i/\sqrt{2}$, then v-dependent contributions to higgs kinetic terms are generated (plus higher dimensional interactions for higgs, involving 2 derivatives). Higgs field re-definitions can be performed to obtain canonical kinetic terms for neutral higgs sector; these bring further corrections to the scalar potential [7] but also shift the pure MSSM part of $\mathcal{L}_\chi$ to generate extra $1/M^2$ corrections. The field re-definitions are then [7]

$$\begin{aligned} h_1^0 &\to h_1^0\left(1-\frac{\tilde g_{11^*}}{2}\right) - \frac{\tilde g_{21^*}}{2}\,h_2^0 \\ h_2^0 &\to h_2^0\left(1-\frac{\tilde g_{22^*}}{2}\right) - \frac{\tilde g_{12^*}}{2}\,h_1^0, \qquad \text{where} \qquad \tilde g_{ij^*} \equiv g_{ij^*}\Big|_{h_i^0\to v_i/\sqrt{2}} \end{aligned} \tag{26}$$

where $\tilde g_{ij^*}$ are constants defined above. This higgs field redefinition is applied to $\mathcal{L}_\chi$ in the doubled-primed basis. The result of applying (23), (26) to (21) is then, after removing the double-primed superscripts:

$$\begin{aligned} \mathcal{L}_\chi &= \delta\mathcal{L}_D + \delta\mathcal{L}_{F,1} + \delta\mathcal{L}_2 \\ &+ \Big\{ \frac{i}{2}\,\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu\psi_1^0 + \frac{i}{2}\;\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu\psi_2^0 + \frac{i}{2}\,\lambda_w^3\sigma^\mu\partial_\mu\overline{\lambda}_w^3 + \frac{i}{2}\,\lambda_y\sigma^\mu\partial_\mu\overline{\lambda}_y \\ &+ \frac{\mu}{2}\,\Big[(-2+\tilde\nu_{11^*}+\tilde\nu_{22^*})\;\psi_1^0\psi_2^0 + \tilde\nu_{3^*4}\,\psi_1^0\psi_1^0 + \tilde\nu_{34^*}\;\psi_2^0\psi_2^0\Big] \\ &+ \frac{g_2}{2\sqrt{2}}\,\lambda_w^3\,\Big[h_1^{0\,*}\,\big[\psi_2^0\,(\tilde\nu_{34^*}-\tilde g_{21^*}) + \psi_1^0\,(-2+\tilde\nu_{11^*}+\tilde g_{11^*}+\tilde\nu_{55^*}^w)\big] \\ &- h_2^{0\,*}\,\big[\psi_1^0\;(\tilde\nu_{3^*4}-\tilde g_{12^*}) + \psi_2^0\,(-2+\tilde g_{22^*}+\tilde\nu_{22^*}+\tilde\nu_{55^*}^w)\big]\Big] - (w\to y, g_2\to g_1) \\ &- \frac{m_1}{2}(1-\tilde\nu_{55^*}^y)\,\lambda_y\lambda_y - \frac{m_2}{2}(1-\tilde\nu_{55^*}^w)\,\lambda_w^3\lambda_w^3 + h.c.\Big\} \end{aligned} \tag{27}$$

This is the canonical Lagrangian in the neutralino sector that contains the mass and kinetic terms. The interaction terms $\mathcal{O}(1/M^2)$ can be found in Appendix B; they are not affected by the above higgs and gaugino/higgsino redefinitions since the difference is of order higher than $1/M^2$. Thus, they can be simply added to the above $\mathcal{L}_\chi$. The full Lagrangian can be implemented in micrOMEGAs, to analyze the impact of $\mathcal{O}_i$, $\mathcal{K}_0$ on dark matter searches.

4 The spectrum of the Lagrangian.

4.1 Corrections to neutralino masses.

Using $\mathcal{L}_\chi$ of the last equation in the previous section, we compute in this section the mass corrections to the neutralino fields, induced by all effective operators in $1/M^2$ order (i.e. leading order in $\alpha_{ij} \sim 1/M^2$, second order in $\zeta_{10} \sim 1/M$). In the basis $(\lambda_y, \lambda_w^3, \psi_1^0, \psi_2^0)^T$ this mass matrix is

$$\begin{aligned}
\mathcal{M}_{11} &= m_1 + \frac{1}{8}\left[-2\,\alpha_{70}^y\,\mu\, v^2 + \left(\alpha_{71}^y\, m_0 + (\alpha_{70}^y + \alpha_{70}^{y*})\; m_1\right) v^2 \sin 2\beta\right], \qquad \mathcal{M}_{12} = 0 \\
\mathcal{M}_{13} &= -\frac{m_Z}{32}\sin\theta_w\Big[-4\left(-8 + (\alpha_{30}+\alpha_{40})\,v^2\right)\cos\beta + v^2\left[4\,(\alpha_{30}+\alpha_{40})\,\cos 3\beta\right. \\
&+ \left. 2\sin\beta\left(4\alpha_{50}^* + 4\alpha_{60} + \alpha_{70}^y + \alpha_{70}^{y\,*} + (4\alpha_{50}^* - 4\alpha_{60} + 3\alpha_{70}^y + \alpha_{70}^{y\,*})\,\cos 2\beta\right)\right]\Big] \\
\mathcal{M}_{14} &= \frac{m_Z}{32}\sin\theta_w\Big[32\sin\beta + 2\,v^2\cos\beta\left[4\,\alpha_{50} + 4\alpha_{60}^* + \alpha_{70}^y + \alpha_{70}^{y\,*}\right. \\
&+ \left.\cos 2\beta\,(4\alpha_{50} - 4\alpha_{60}^* - 3\alpha_{70}^y - \alpha_{70}^{y\,*}) - 4\,(\alpha_{30}+\alpha_{40})\,\sin 2\beta\,\right]\Big] \\
\mathcal{M}_{22} &= m_2 + \frac{1}{8}\left[-2\alpha_{70}^w\,\mu\, v^2 + \left(\alpha_{71}^w\, m_0 + (\alpha_{70}^w + \alpha_{70}^{w\,*})\, m_2\,\right) v^2\sin 2\beta\,\right] \\
\mathcal{M}_{23} &= \frac{m_Z}{32}\cos\theta_w\Big[-4\left(-8 + (\alpha_{30}+\alpha_{40})\,v^2\right)\cos\beta + v^2\left[4\,(\alpha_{30}+\alpha_{40})\,\cos 3\beta\right. \\
&+ \left. 2\left(4\alpha_{50}^* + 4\alpha_{60} + \alpha_{70}^w + \alpha_{70}^{w\,*} + (4\;\alpha_{50}^* - 4\alpha_{60} + 3\alpha_{70}^w + \alpha_{70}^{w*})\,\cos 2\beta\,\right)\sin\beta\;\right]\Big] \\
\mathcal{M}_{24} &= \frac{m_Z}{32}\cos\theta_w\Big[-32\sin\beta - 2\,v^2\,\cos\beta\left[\,4\alpha_{50} + 4\,\alpha_{60} + \alpha_{70}^w + \alpha_{70}^{w\,*}\right. \\
&+ \left.(4\,\alpha_{50} - 4\,\alpha_{60}^* - 3\alpha_{70}^w - \alpha_{70}^{w*})\,\cos 2\beta - 4\,(\alpha_{30}+\alpha_{40})\sin 2\beta\;\right]\Big] \\
\mathcal{M}_{33} &= \frac{1}{4}v^2\Big[4\,m_0\,\cos\beta\,(\alpha_{11}^*\cos\beta - \alpha_{51}^*\,\sin\beta) + 4\,\zeta_{10}\,\sin^2\beta + \mu\left[2\alpha_{50} + \alpha_{50}^* + \alpha_{60}\right. \\
&+ \left.(-2\alpha_{50} + \alpha_{50}^* - \alpha_{60})\,\cos 2\beta - (4\,\alpha_{10} + \alpha_{30} + \alpha_{40})\,\sin 2\beta\right]\Big] \\
\mathcal{M}_{34} &= \frac{1}{4}\Big[4\,\mu - \Big(2\,(\alpha_{51}^* + \alpha_{61}^*)\,m_0 + \left(2\alpha_{10} + 2\alpha_{20} + 3\,(\alpha_{30}+\alpha_{40})\right)\mu\Big)\,v^2 \\
&+ v^2\Big(2\,\left(-(\alpha_{51}^* - \alpha_{61}^*)\,m_0 - \mu\,(\alpha_{10} - \alpha_{20})\,\right)\,\cos 2\beta \\
&+ \left[(\alpha_{31}^* + \alpha_{41}^* + 2\,\alpha_{81}^*\,)\;m_0 + (3\,\alpha_{50} + \alpha_{50}^* + 3\,\alpha_{60} + \alpha_{60}^*)\,\mu + 4\zeta_{10}\right]\,\sin 2\beta\Big)\Big] \\
\mathcal{M}_{44} &= \frac{1}{4}v^2\Big[2\alpha_{21}^*\, m_0 + (\alpha_{50} + 2\,\alpha_{60} + \alpha_{60}^*)\mu + \left[-2\alpha_{21}^* m_0 + (\alpha_{50} + 2\alpha_{60} - \alpha_{60}^*)\,\mu\right]\cos 2\beta \\
&+ 4\,\zeta_{10}\,\cos^2\beta - \left[2\alpha_{61}^*\, m_0 + \mu\,(4\,\alpha_{20} + \alpha_{30} + \alpha_{40})\,\right]\sin 2\beta\Big]
\end{aligned} \tag{28}$$

with the remaining matrix elements fixed by the symmetry $\mathcal{M}_{ij} = \mathcal{M}_{ji}$.

One can find an eigenvalue (denoted ξ) of this mass matrix in an analytical approach, by a perturbative method, as an expansion about the corresponding MSSM eigenvalue (ξ_o). In

both cases, the eigenvalues satisfy a characteristic equation

$$\gamma_l\, \xi^l = 0, \quad (a) \qquad\qquad \gamma_l^0\, (\xi_o)^l = 0, \quad (b) \tag{29}$$

with sums understood over the repeated index $l = 0, 1, 2, 3, 4$. Here (a) refers to the general case and (b) to the MSSM case. γ_l are coefficients depending on $\alpha_{ij} \sim 1/M^2$ and $\zeta_{10} \sim 1/M$ that are found from the mass matrix above and ξ denotes any of the four mass eigenvalues in the general case. The values of the MSSM counterparts, γ_l^0 for coefficients and ξ_o for corresponding eigenvalue, are recovered from the general ones by setting α_{ij} and ζ_{10} to 0. Further, any general mass eigenvalue can be written $\xi = \xi_o + z_1 + z_2$ with $z_1 \propto \zeta_{10} = \mathcal{O}(1/M)$ due to the $d = 5$ operator, and $z_2 \propto \sum_{k=1,..8} \sum_{ij} \alpha_{ij}\, \sigma_{kij} + \beta_2\, \zeta_{10}^2 = \mathcal{O}(1/M^2)$, due to all $d = 6$ operators as well as the $d = 5$ operator. From these equations, one computes the difference $\xi - \xi_o$ by consistently retaining the leading order approximation in eq.(29) (a). One finds

$$\xi = \xi_o + z_1 - \frac{1}{j\, \gamma_j^0\, (\xi_o)^{j-1}} \Big[\gamma_k^{(2)}\, (\xi_o)^k + z_1^2\, C_k^2\, \gamma_k^0\, (\xi_o)^{k-2} + z_1\, k\, \gamma_k^{(1)}\, (\xi_o)^{k-1}\Big] \tag{30}$$

where

$$z_1 = -\frac{\gamma_k^{(1)}\, (\xi_o)^k}{j\, \gamma_j^0\, (\xi_o)^{j-1}} \tag{31}$$

with summations understood over indices $j, k = 0, 1, 2, 3, 4$. Here $\gamma_k^{(1)}$ and $\gamma_k^{(2)}$ denote the corrections $\mathcal{O}(1/M)$ and $\mathcal{O}(1/M^2)$ respectively, that are present in γ_k. Also $C_k^2 = k(k-1)/2$. Replacing ξ_o by any of the four values of the neutralino masses in the MSSM, one obtains the corresponding neutralino mass in the general case. In particular this is true about the LSP mass, when ξ_o is the MSSM corresponding eigenvalue.

We provide below the analytical expression for the neutralino mass corrections, with the contribution of each operator ($\mathcal{O}_i$) labelled by the first index in α_{ij}. One can include the effect of a selected set of these operators or from all of them, by simply adding the *corrections* $\delta m_\chi = \xi - \xi_o$ to the MSSM mass eigenvalue (ξ_o), due to the particular set of operators considered. We found

$$\delta m_\chi = \xi - \xi_o = \sum_i \delta m_\chi(\mathcal{O}_i) + \delta m_\chi(\mathcal{K}_0) \tag{32}$$

$$\begin{aligned}
\delta m_\chi(\mathcal{O}_1) &= \frac{2\alpha_{10}}{\sigma}\, \mu\, v^2\, \cos\beta \Big[2\, \mu\, (m_1 - \xi_o)\, (\xi_o - m_2)\, \cos\beta - \big[(m_1 + m_2)\, m_Z^2 \\
&+ 2\, (m_1\, m_2 - m_Z^2)\, \xi_o - 2\, (m_1 + m_2)\, {\xi_o}^2 + 2\, {\xi_o}^3 + (m_1 - m_2)\, m_Z^2\, \cos 2\theta_w\big] \sin\beta\Big] \\
&+ \frac{\alpha_{11}^*}{4\, \sigma}\, m_0\, v^2\, \Big[2 \cos^2\beta \big[(m_1 + m_2)\, m_Z^2 - 2\, (-2 m_1 m_2 + m_Z^2)\, \xi_o - 4\, (m_1 + m_2)\, {\xi_o}^2 \\
&+ 4\, {\xi_o}^3 - m_Z^2\, (m_1 + m_2 - 2\, \xi_o)\, \cos 2\beta\big] + (m_1 - m_2)\, m_Z^2\, \cos 2\theta_w \sin^2 2\beta\Big]
\end{aligned} \tag{33}$$

where σ is defined later on. Further:

$$\begin{aligned}
\delta m_\chi(\mathcal{O}_2) &= \delta m_\chi(\mathcal{O}_1)\Big[\alpha_{10} \to \alpha_{20}, \alpha_{11}^* \to \alpha_{21}^*, \beta \to \pi/2 - \beta\Big] \\
\delta m_\chi(\mathcal{O}_3) &= \frac{\alpha_{30}}{4\sigma}\, v^2 \Big[- 12 m_1\, m_2\, \mu^2 + (m_1 + m_2)(12\mu^2 + m_Z^2)\, \xi_o - 2\,(6\mu^2 + m_Z^2)\, {\xi_o}^2 \\
&+ m_Z^2\, \xi_o\, (2\,\xi_o - m_1 - m_2) \cos 4\beta + 2 \sin 2\beta\, \big[\mu\, \big(- 3\,(m_1 + m_2)\, m_Z^2 - 2\, m_1\, m_2\, \xi_o \\
&+ 6\, m_Z^2\, \xi_o + 2\,(m_1 + m_2)\, {\xi_o}^2 - 2\,{\xi_o}^3\big) + m_Z^2\,(m_1 - m_2) \cos 2\theta_w\, (\xi_o \sin 2\beta - 3\,\mu)\big]\Big] \\
&+ \frac{\alpha_{31^*}}{4\sigma}\, m_0\, v^2\, \sin 2\beta \Big[m_Z^2\, \big(m_1 {+} m_2 - 2\,\xi_o + (m_1 - m_2)\, \cos 2\theta_w\,\big) \sin 2\beta \\
&+ 4\,\mu(m_1 - \xi_o)(m_2 - \xi_o)\Big] \qquad (34)
\end{aligned}$$

and

$$\delta m_\chi(\mathcal{O}_4) = \delta m_\chi(\mathcal{O}_3)\Big[\alpha_{30} \to \alpha_{40}, \alpha_{31} \to \alpha_{41}\Big] \qquad (35)$$

$$\begin{aligned}
\delta m_\chi(\mathcal{O}_5) &= \frac{\alpha_{50}^*}{8\sigma} v^2 \cos\beta \Big[- 5\mu\, m_Z^2\, (m_1 + m_2 - 2\,\xi_o)\, \cos 3\beta + 2\, \sin\beta\, \big[8\, m_1\, m_2\, \mu^2 \\
&- (m_1 {+} m_2)(8\mu^2 {+} m_Z^2)\xi_o + 2(4\mu^2 {+} m_Z^2){\xi_o}^2\big] + 8(m_2 {-} m_1) m_Z^2 \xi_o \cos^2\beta \cos 2\theta_w \sin\beta \\
&+ \mu \cos\beta\, \big[5\,(m_1 + m_2)\, m_Z^2 + 2(4 m_1 m_2 - 5 m_Z^2)\, \xi_o - 8(m_1 + m_2)\, {\xi_o}^2 + 8\,{\xi_o}^3 \\
&+ 20\,(m_1 - m_2)\, m_Z^2 \cos 2\theta_w \sin^2\beta\big] - 2\, m_Z^2\, (m_1 + m_2 - 2\,\xi_o)\, \xi_o \sin 3\beta\Big] \\
&+ \frac{\alpha_{50}}{16\sigma}\, v^2 \Big[- \mu m_Z^2 \cos 4\beta\, \big[m_1 + m_2 - 2\,\xi_o + (m_1 - m_2) \cos 2\theta_w\big] \\
&+ 4\mu \cos 2\beta\, \big[(m_1 + m_2)\, m_Z^2 - 2(m_1 m_2 + m_Z^2)\, \xi_o + 2(m_1 + m_2)\, {\xi_o}^2 - 2{\xi_o}^3 \\
&+ (m_1 - m_2)\, m_Z^2 \cos 2\theta_w\big] + 3\mu\big[7\,(m_1 + m_2)\, m_Z^2 + 2(4\, m_1 m_2 - 7 m_Z^2)\, \xi_o \\
&- 8(m_1 + m_2)\, {\xi_o}^2 + 8{\xi_o}^3 + 7(m_1 - m_2)\, m_Z^2 cos2\theta_w\big] - 4\, \sin 2\beta\big[- 12 m_1 m_2 \mu^2 \\
&+ (m_1 + m_2)(12\mu^2 + m_Z^2)\, \xi_o - 2\,(6\mu^2 + m_Z^2)\, {\xi_o}^2 + (m_1 - m_2)\, m_Z^2 \xi_o\, \cos 2\theta_w\big] \\
&- 2 m_Z^2\, \xi_o\, (m_1 + m_2 - 2\xi_o + (m_1 - m_2)\, \cos 2\theta_w)\, \sin 4\beta\Big] + \frac{\alpha_{51}^*}{8\sigma}\, m_0\, v^2 \\
&\times \Big[32\mu(m_1 - \xi_o)(\xi_o - m_2)\, \cos^2\beta - 2 \sin 2\beta\big[3(m_1 + m_2) m_Z^2 + 4 m_1 m_2\, \xi_o - 6 m_Z^2\, \xi_o \\
&- 4\,(m_1 + m_2)\, {\xi_o}^2 + 4{\xi_o}^3 + 3(m_1 - m_2)\, m_Z^2\, \cos 2\theta_w\big] - m_Z^2\, \big[m_1 + m_2 - 2\,\xi_o \\
&+ (m_1 - m_2) \cos 2\theta_w\big] \sin 4\beta\Big] \qquad (36)
\end{aligned}$$

Further

$$\delta m_\chi(\mathcal{O}_6) = \delta m_\chi(\mathcal{O}_5)\Big[\alpha_{50} \to \alpha_{60}, \alpha_{51} \to \alpha_{61}, \beta \to \pi/2 - \beta\Big] \qquad (37)$$

The correction due to bino part (indexed by y) of $\mathcal{O}_7$ is:

$$
\begin{aligned}
\delta m_\chi(\mathcal{O}_7^y) &= \frac{\alpha_{70}^y}{16\sigma}\, v^2 \Big[\mu \left[m_1\, m_Z^2 + 8\mu^2\, \xi_o + 5\, m_Z^2\, \xi_o - 8\, {\xi_o}^3 - m_2\, (8\mu^2 + m_Z^2 - 8\, {\xi_o}^2)\right] \\
&+ \mu\, m_Z^2 \left[(m_1 + m_2 + 3\, \xi_o)\, \cos 2\theta_w - \cos 4\beta \,\big(m_1 + 3\, m_2 - 3\, \xi_o \right. \\
&+ \left.(m_1 - 3\, m_2 + 3\, \xi_o)\, \cos 2\theta_w\big)\right] - 2\big[m_Z^2\, (2\mu^2 + (m_2 - \xi_o)\, \xi_o) + m_1\, \big(- 2m_2\, \mu^2 \\
&+ \xi_o\, (2\mu^2 + m_Z^2) + 2m_2{\xi_o}^2 - 2\, {\xi_o}^3\big) + m_Z^2 \cos 2\theta_w \big(2\mu^2 + \xi_o(m_1 - m_2 + \xi_o)\big)\big] \sin 2\beta\Big] \\
&+ \frac{\alpha_{70}^{y\,*}}{8\sigma_1} v^2 \sin 2\beta \Big[2m_1 m_2 \mu^2 - \big(2m_1\mu^2 + (m_1+m_2) m_Z^2\big)\, \xi_o + (-2m_1 m_2 + m_Z^2){\xi_o}^2 \\
&+ 2\, m_1 {\xi_o}^3 + m_Z^2 \big(\mu(m_1 + m_2 - \xi_o) \sin 2\beta + (m_1 - m_2 + \xi_o) \cos 2\theta_w (-\xi_o + \mu \sin 2\beta)\big)\Big] \\
&+ \frac{1}{8\sigma}\, \alpha_{71}^y\, m_0\, v^2 \sin 2\beta \,\Big[2\, m_2(\mu^2 - {\xi_o}^2) - \xi_o\, (2\mu^2 + m_Z^2 - 2\, {\xi_o}^2) \\
&+ m_Z^2\, (-\xi_o\, \cos 2\theta_w + 2\mu \cos^2 \theta_w \sin 2\beta)\,\Big] \qquad (38)
\end{aligned}
$$

A similar correction exists for $\mathcal{O}_7^w$:

$$
\delta m_\chi(\mathcal{O}_7^w) = \delta m_\chi(\mathcal{O}_7^y) \Big[\alpha_{70}^y \to \alpha_{70}^w, \alpha_{71}^y \to \alpha_{71}^w, m_1 \to m_2, m_2 \to m_1, \theta_w \to \pi/2 - \theta_w\Big] \qquad (39)
$$

Further

$$
\begin{aligned}
\delta m_\chi(\mathcal{O}_8) &= \frac{\alpha_{81}^*}{2\sigma}\, m_0\, v^2 \sin 2\beta \,\Big[4\mu(m_1 - \xi_o)(m_2 - \xi_o) + m_Z^2 [m_1 + m_2 - 2\xi_o \\
&+ (m_1 - m_2) \cos 2\theta_w] \sin 2\beta\Big] \qquad (40)
\end{aligned}
$$

Finally

$$
\begin{aligned}
\delta m_\chi(\mathcal{K}_0) &= \frac{\zeta_{10}}{4\sigma}\, v^2 \sigma' + \frac{\zeta_{10}^2}{8\, \sigma^3}\, v^4 \Big[12\, \sigma^2\, (m_1 - \xi_o)(m_2 - \xi_o)\, \sin^2 2\beta + \sigma'^2 \big[- m_1 m_2 + \mu^2 \\
&+ m_Z^2 + 3(m_1 + m_2)\, \xi_o - 6\, {\xi_o}^2\big] + \sigma\sigma' \,\big[4\, m_1\, m_2 - 5m_Z^2 - 8(m_1 + m_2)\, \xi_o + 12{\xi_o}^2 \\
&+ m_Z^2 \cos 4\beta - 8\mu(m_1 + m_2 - 2\, \xi_o) \sin 2\beta\big]\Big] \qquad (41)
\end{aligned}
$$

In the above equations we introduced the notation

$$
\begin{aligned}
\sigma &= (m_1 + m_2)(2\mu^2 + m_Z^2) - 4\, (-m_1\, m_2 + \mu^2 + m_Z^2)\, \xi_o - 6\, (m_1 + m_2)\, {\xi_o}^2 + 8\, {\xi_o}^3 \\
&+ m_Z^2 \left[(m_1 - m_2) \cos 2\theta_w + 2\mu \sin 2\beta\right] \\
\sigma' &= 5\, (m_1 + m_2)\, m_Z^2 + 2(4\, m_1\, m_2 - 5\, m_Z^2)\, \xi_o - 8\, (m_1 + m_2)\, {\xi_o}^2 + 8\, {\xi_o}^3 \\
&+ m_Z^2 \big[- \big((m_1 + m_2 - 2\, \xi_o) \cos 4\beta\big) - (m_1 - m_2)(-5 + \cos 4\beta) \cos 2\theta_w\big] \\
&+ 16\mu\, (m_1 - \xi_o)(m_2 - \xi_o) \sin 2\beta \qquad (42)
\end{aligned}
$$

In some cases not all operators are present, and even for each operator, one may be interested only in the supersymmetric correction (labelled by a "zero" second index, α_{j0}). By simply setting α_{j1} and α_{j1}^* to zero, one can identify only the supersymmetric corrections, when the

above results simplify considerably. Moreover, for the constrained MSSM when the gaugino universality is present, $m_1(m_Z) = (5/3)\tan^2\theta_w(m_Z) m_2(m_Z)$, then the results are further simplified. As for the expression of the MSSM mass eigenvalues denoted ξ_o, these are known in the literature [28] and can also be evaluated numerically. The numerical results due to these corrections will be presented in Section 5.

4.2 Corrections to the Higgs fields masses.

The effective operators also affect the spectrum in the Higgs sector. The exact corrections of these operators to lightest Higgs mass m_h^2 to order $\mathcal{O}(1/M^2)$ can be found in eq.(36) and Appendix C of [7]. With $m_A > m_Z$ assumed, the mass correction to m_h, in the large $\tan\beta$ limit with m_A fixed, has a very simple form [7]:

$$
\begin{aligned}
\delta m_h^2 &= -2\,v^2\Big[\alpha_{22}\,m_0^2+(\alpha_{30}+\alpha_{40})\mu^2+2\alpha_{61}\,m_0\,\mu-\alpha_{20}\,m_Z^2\Big]-\frac{(2\,\zeta_{10}\,\mu)^2\;v^4}{m_A^2-m_Z^2}\\
&+\frac{v^2}{\tan\beta}\bigg[\frac{1}{(m_A^2-m_Z^2)}\Big(4\,m_A^2\,\big(\,(2\alpha_{21}+\alpha_{31}+\alpha_{41}+2\alpha_{81})\,m_0\,\mu+(2\alpha_{50}+\alpha_{60})\,\mu^2+\alpha_{62}\,m_0^2\big)\\
&-\;(2\alpha_{60}-3\alpha_{70})\,m_A^2\,m_Z^2-(2\alpha_{60}+\alpha_{70})\,m_Z^4\Big)+\frac{8\,(m_A^2+m_Z^2)\;(\mu\,m_0\,\zeta_{10}\,\zeta_{11})\,v^2}{(m_A^2-m_Z^2)^2}\bigg]\\
&+\;\mathcal{O}(1/\tan^2\beta)
\end{aligned}
\tag{43}
$$

where $\delta m_h^2 \equiv m_h^2 - m_h^{0\,2}$, with m_h^0 the MSSM value. Similar formulae exist for the heavier neutral Higgs mass m_H^2, and pseudoscalar m_A^2. With these corrections one can examine the effect of combined dark matter and EW constraints on the scale of new physics that may be present in the MSSM. For details on the Higgs sector corrections to masses see [7, 8]. In the numerical results presented in Section 5 we use the exact formula for δm_h^2 (i.e. with no expansion in $\tan\beta$ or other parameter). Eq.(43) gives however a good indication on the behaviour of the corrections: for example δm_h^2 is increased by negative α_{30}, α_{40}, as we shall see later on, α_{50}, α_{60} have contributions suppressed at large $\tan\beta$, Susy breaking corrections $(\alpha_{j1}, \alpha_{j2})$ are comparable to Susy ones (α_{j0}), etc.

4.3 Corrections to the chargino masses.

Following the method presented for the neutralino case, one can also evaluate the corrections to the chargino fields masses. To this purpose one uses the Lagrangian presented in Appendix B. As for the neutralino case, chargino fields rescaling is required to ensure canonical kinetic terms for them, plus Higgs re-definitions as in eq.(26). The chargino mass corrections in order $1/M^2$ are presented in Appendix C and can be useful for phenomenological studies, in a global fit of MSSM with effective operators.

5 Phenomenological implications

In this section we analyze the impact of the effective operators on the MSSM Higgs and neutralino LSP masses. We perform the analysis separately for each individual operator considered. This is because not all operators may be present in a particular model. The corrections from a set of operators can be readily obtained by combining appropriately those of individual $\mathcal{O}_i$. The number of parameters can be further reduced by considering only the impact of the supersymmetric corrections (i.e. take $\alpha_{j1} = \alpha_{j2} = 0$, $j = 1, ...8$), and $\alpha_{j0} \neq 0$. It turns out that non-Susy corrections are, in absolute value, of a size generically close to that of the supersymmetric case. From this analysis one can identify which of these operators has the largest impact on phenomenology.

To this purpose we consider the CMSSM phase space points that respect all current constraints, both theoretical and experimental. These refer to: radiative EWSB, no electric charge or colour breaking, LEP sparticle bounds, $b \to s\gamma$ bounds and dark matter constraints, but no LEP2 bound on the Higgs mass (that is not imposed, see later). These points are selected using SOFTSUSY [27] and micrOMEGAs [24] codes, in the context of CMSSM, as described and used in [3]. On these phase space points we impose the constraint $\tilde{m}/M < 1/2$, where $\tilde{m} = \mu, m_0$, or m_{12}, to ensure that our effective expansion parameter which is actually its square $\alpha_{j0}\tilde{m} \sim (\tilde{m}/M)^2 < 1/4$. This value is small enough to trust the results of the effective operators expansion. Using these CMSSM phase points we examine the effect on the LSP and Higgs masses from the corrections due to each effective operator. That is, we treat the effective operators as a perturbation of the CMSSM "background", to analyze which of its phase space points are likely to give sizable corrections. In this way we investigate if the "best fit" points of CMSSM are stable under corrections from the effective operators. By "best fit" points here we mean those points that satisfy the aforementioned theoretical and experimental constraints, with WMAP dark matter relic density consistency or saturation within 3σ, plus electroweak fine tuning[4] not worse than 1 part in 200 [3]. This fine tuning constraint enforces that some points have an expansion parameter less than 1/4.

A more careful analysis should implement all the new couplings in the Higgs, chargino and neutralino sectors in SOFTSUSY and micrOMEGAs codes, to evaluate the impact of these operators. One could then find bounds on the effective operators coefficients that can be translated into upper bounds on the corrections to the MSSM Higgs mass.

[4]The definition of EW scale fine tuning that we are using is

$$\Delta \equiv \max \left|\Delta_p\right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \qquad \Delta_p \equiv \frac{\partial \ln v^2}{\partial \ln p} \tag{44}$$

where p are input parameters at the UV scale, in the standard MSSM notation. For its value at 2-loop see [3].

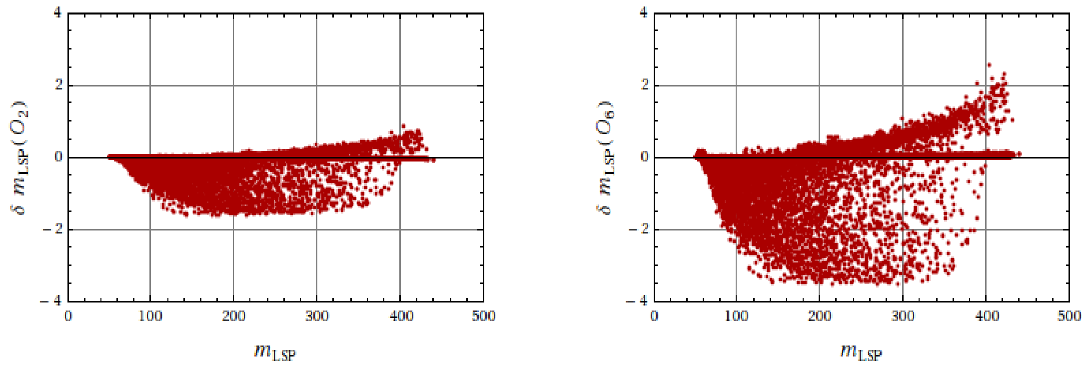

Figure 1: The corrections to the LSP mass, induced by O_2 and O_6 respectively. These are the $d = 6$ operators that bring the largest corrections δm_{LSP} and are generated for a scale of new physics of $M = 8$ TeV. The corrections are for $\alpha_{j0} = \alpha_{j1} = -1/M^2$, $j = 2, 6$, and include Susy breaking effects.

5.1 The neutralino sector.

Let us now discuss the mass corrections to the neutralino LSP. A numerical analysis of the results found in Section 4.1 shows that the mass corrections are actually very small. In Figure 1 we showed the mass corrections induced by operators $\mathcal{O}_2$ and $\mathcal{O}_6$ which bring the largest corrections, as a function of m_{LSP} in CMSSM for $M = 8$ TeV, value consistent with ρ-parameter constraints [14]. As shown, the mass corrections to neutralino LSP are of the order of few GeV only (including non-Susy corrections), while for the remaining operators, these corrections are even smaller. The reason for this is that the mass of the LSP is suppressed - at large μ - not only by α_{ij}, but also by large μ. The mass corrections increase slightly when non-supersymmetric effects are also included, accounted for by α_{j1} in the formulae of Section 4.1. Given that the correction to the LSP mass eigenvalue is so small, the LSP composition cannot then be significantly changed from its CMSSM value; for example the variation of $|\langle\chi|u_i\rangle|^2 - |\langle\chi^0|u_i\rangle|^2$, where χ (χ^0) is the neutralino LSP in the presence (absence) of effective operators, and $u_i = \lambda_y, \lambda_w^3, \psi_1, \psi_2$, does not change by more than $\pm 1\%$ (for all i), for $M = 8$ TeV, which is very small. The presence of a set or all $d = 6$ operators can however increase the overall effect on m_{LSP} and LSP composition, but this depends on the relative signs of the operators and is not studied here. For a detailed study of the neutralino sector in the presence of the $d = 5$ operator see [15, 16, 17].

5.2 The Higgs sector.

Let us now discuss the corrections to the mass m_h of the lightest MSSM Higgs field. In [7] analytical corrections to m_h from effective operators were computed in $1/M^2$ order. This was followed by a simple estimate of the overall size of the correction to tree-level m_h, in a very special case and under simplifying assumptions for the coefficients of the operators. In this

section we improve the numerical analysis, to present a general and accurate numerical investigation of the corrections to m_h for *individual* operators and including *quantum corrections*, not considered before.

The results are illustrated in the plots of Figures 2,3,4 where the *supersymmetric* correction δm_h is shown for each operator $\mathcal{O}_{1,...6}$ and $\mathcal{K}_0$ as a function of the CMSSM value for m_h evaluated at 2-loop leading-log (LL), for $M = 10$ and 8 TeV (values consistent with ρ-parameter constraints [14]). The correction due to $\mathcal{O}_7$ is very small (< 1GeV) for all the parameter space, because it is strongly suppressed by the small gauge couplings, in addition to α_{ij}, and it is not shown here. The correction δm_h shown in the plots as a function of CMSSM value of m_h, is defined as

$$\delta m_h = \left[m_h^2\big|_{2-\text{loop,MSSM}} + \delta m_h^2 \right]^{1/2} - m_h\big|_{2-\text{loop,MSSM}} = \frac{1}{2} \frac{\delta m_h^2}{m_h\big|_{2-\text{loop,MSSM}}} + \mathcal{O}(1/M^4) \quad (45)$$

so the total value is then $\delta m_h + m_h|_{2-\text{loop,MSSM}}$; here $m_h|_{2-\text{loop,MSSM}}$ is the 2-loop (LL) corrected CMSSM value for Higgs mass, while δm_h^2 is the classical correction due to the effective operators whose exact expression can be found, for exact Susy case, in eq.(36) in [7]. The large $\tan\beta$ limit of δm_h^2 is given in (43), including Susy breaking effects; as it can be seen from there, negative α_{30} and α_{40} can bring a positive correction δm_h, and this remains true for all $\tan\beta$ as seen in the plots in Figure 2, 3; for α_{j0}, $j = 1, 2, 5, 6$ the sign of the correction is not clear from (43). Finally, in all plots the points below the black continuous line are the CMSSM points with EW fine tuning $\Delta < 200$, and satisfy all experimental and theoretical constraints as explained above, including WMAP constraint within 3σ (in red) or consistent with it (in blue), except the LEP2 bound on m_h which is never imposed, for reasons that become clear below.

Let us first discuss the correction for points with $\Delta < 200$. From these plots we notice that CMSSM ("best fit") points below the black continuous line, i.e. which respect all constraints mentioned above plus fine tuning $\Delta < 200$ and regardless of the LEP2 bound on m_h, receive from individual operators a small change to the Higgs mass m_h, of only few GeV: up to 4 GeV for $M = 10$ TeV and up to 6 GeV for $M = 8$ TeV. This indicates a variation of δm_h by about 1 GeV for a 1 TeV variation of M. These numerical values are even smaller for some operators, see Figures 2,3. Note that points which were below the LEP2 bound by this correction are now phenomenologically viable. The special point of CMSSM of minimal $\Delta = 18$ that saturates the dark matter relic density [5] within 3σ, and with $m_h = 115.9 \pm 2$ GeV [3], could therefore receive a correction $\delta m_h \sim 4$ to 6 GeV, so that m_h can increase to $m_h + \delta m_h = (120-122) \pm 2$ GeV. Given the relatively small size of the correction δm_h one can say that these particular CMSSM phase space points and their predictions are stable against the presence of new physics at the scale $M = 10$ TeV or $M = 8$ TeV. This is an interesting finding, and can be explained by the fact that these points generically have a light μ and

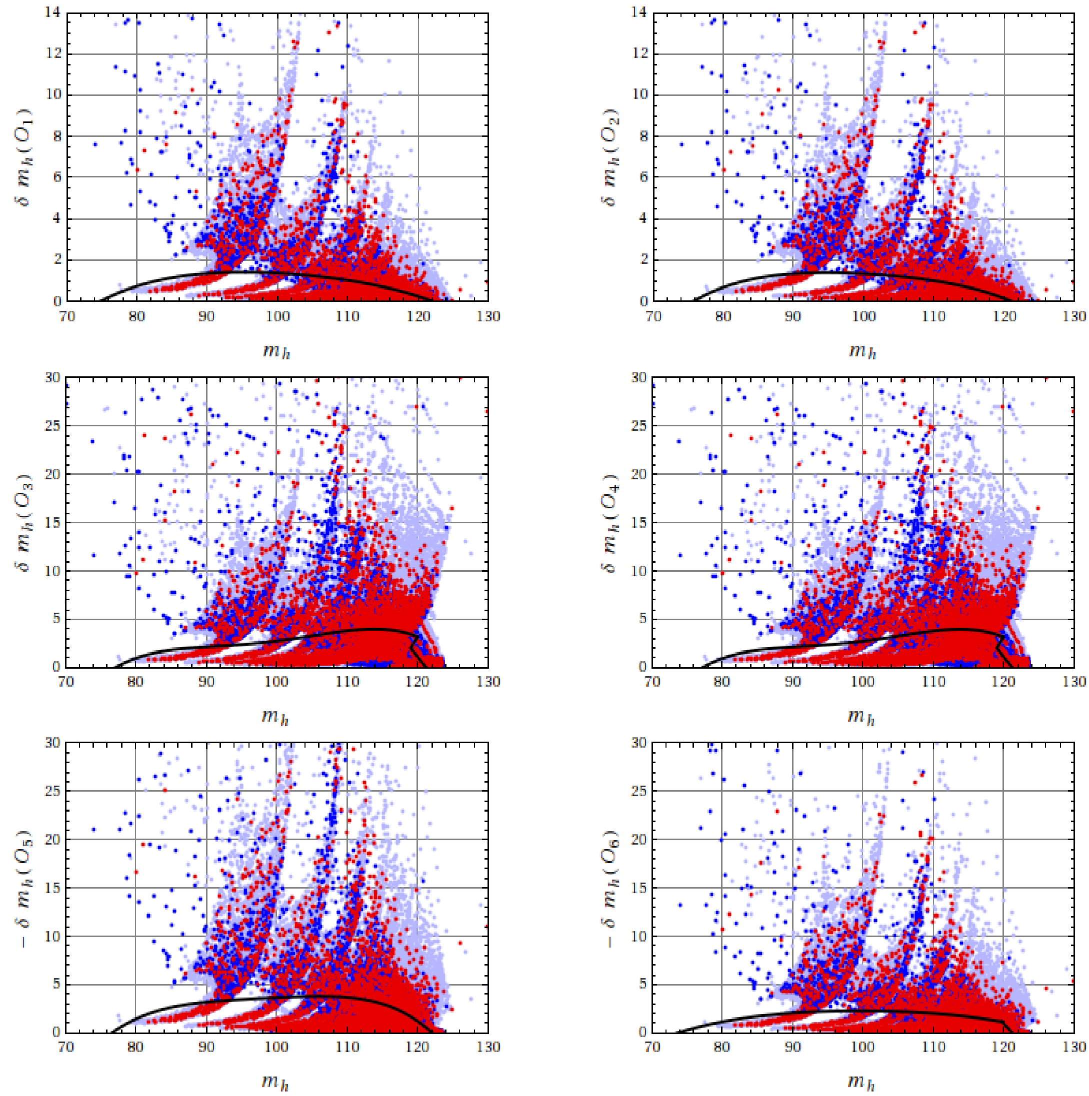

Figure 2: The correction δm_h to the lightest MSSM Higgs mass, due to effective operators, as a function of the 2-loop (LL) CMSSM mass m_h, with $M = 10$ TeV. In light blue are CMSSM phase space points with relic density $\Omega h^2 \geq 0.1285$; on top, in dark blue, are points with $\Omega h^2 \leq 0.0913$ (3σ deviation) and on top, in red, are MSSM points that saturate WMAP bound within 3σ: $\Omega h^2 = 0.1099 \pm 0.0186$. (WMAP value: $\Omega h^2 = 0.1099 \pm 0.0062$ [5]). No LEP2 bound on m_h is imposed at any time. The corrected value of the Higgs mass is $m_h + \delta m_h$. The corrections are supersymmetric, generated by α_{j0}, and can increase/decrease if Susy-breaking effects $(\alpha_{j1}, \alpha_{j2})$ are also included. We assumed $\alpha_{j0} = -1/M^2$, $j = 1, 2, ..6$ and $M = 10$ TeV. The points below (above) the black continuous line have CMSSM EW fine-tuning $\Delta < 200$ ($\Delta > 200$), respectively. The points below the continuous line receive a correction of up to 4 GeV. With α_{50}, $\alpha_{60} < 0$, $\delta m_h(\mathcal{O}_{5,6}) < 0$ (note that $-\delta m_h(\mathcal{O}_{5,6})$ is plotted). The gaps ("wedges") in the plots would be filled in by a better scan of the phase space.

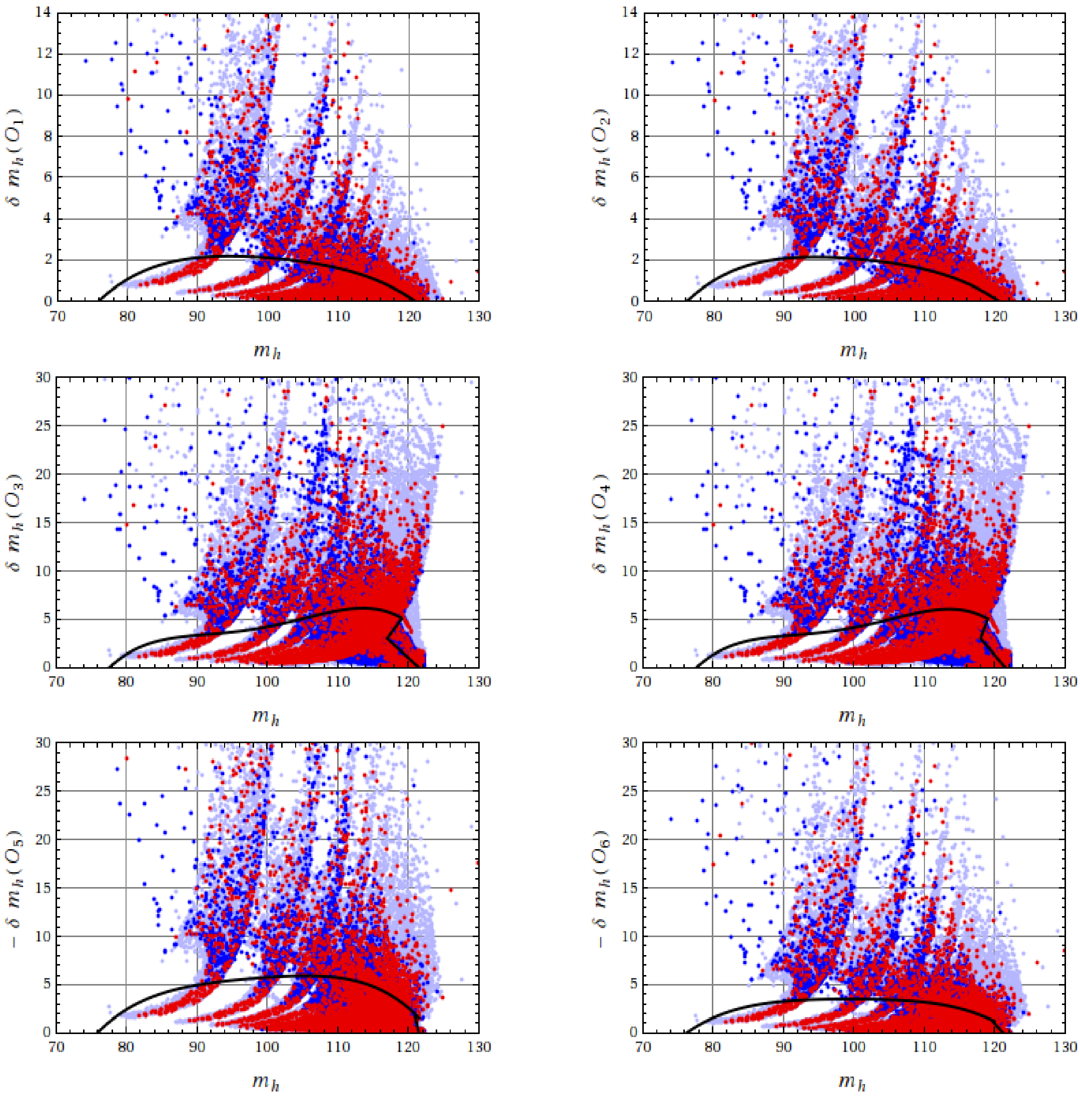

Figure 3: As for Figure 2, but with M=8 TeV. The continuous line of $\Delta = 200$ has changed position and points under it can bring a δm_h up to 6 GeV.

light m_{12} (focus point region) [3], and thus the supersymmetric corrections δm_h, (generated by α_{j0}) are rather suppressed.

The corrections δm_h can increase or decrease if one also includes effects of Susy breaking associated with the effective operators and encoded in α_{j1}, α_{j2}, by an amount comparable to that due to their supersymmetric corrections; for large $\tan\beta$ the size of their effects can also be seen from eq.(43). However the relevance of such corrections for the little hierarchy problem and for m_h value is questionable, given that these are themselves related to supersymmetry breaking. We therefore do not consider such effects further. Finally, unlike operators $\mathcal{O}_i$, in the case of the dimension-five operator, the MSSM phase space points of $\Delta < 200$ that violate

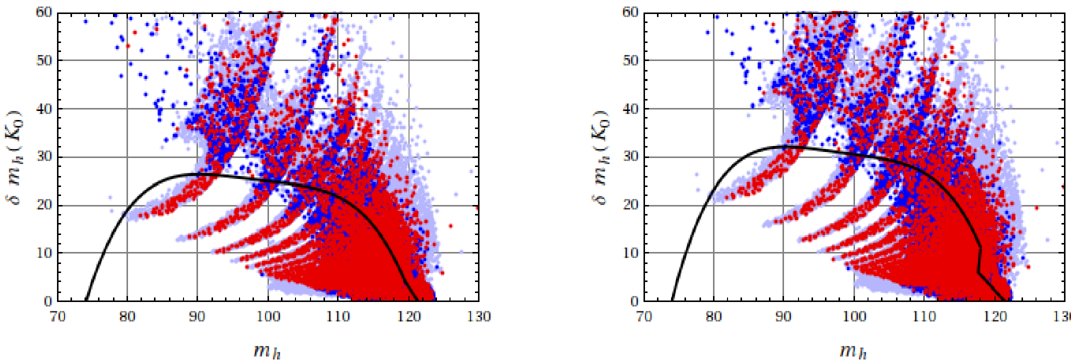

Figure 4: As for Figure 2, with the correction to the MSSM lightest Higgs mass induced by $\mathcal{K}_0$, in function of 2-loop MSSM Higgs mass. Left plot: $M = 10$ TeV, right plot: $M = 8$ TeV. The corrections from $d = 5$ operators are now larger, due to leading $1/M$ terms present for the $d = 5$ operator.

the LEP2 bound can bring a correction $\delta m_h(\mathcal{K}_0)$ significantly larger, and these points become phenomenologically viable (see Figure 4, with $\delta m_h \sim 20$ GeV for $m_h \sim 110$ GeV, still under the black curve, i.e. $\Delta < 200$); however, the final, corrected value $m_h + \delta m_h$, while situated above the LEP2 bound now, is still below 125-135 GeV.

Based on our previous results for the corrections to the neutralino LSP mass, which turned out to be significantly smaller, we can say that these "best fit" CMSSM phase space points ($\Delta < 200$) are unlikely to have their dark matter constraint changed significantly, and are then rather stable under "new physics" presence. This is also supported by the fact that dark matter abundance, that depends on the annihilation cross section may not receive large corrections since the change of the LSP composition due to $\mathcal{O}_i$ was small (consistent with a small mass correction). However, only a careful implementation of the new couplings in the neutralino sector into micrOMEGAs and SOFTSUSY can address this issue on solid grounds. Note that to such cross section effects all operators $\mathcal{O}_i$ contribute: some like $\mathcal{O}_7$ provide a direct LSP annihilation coupling of the bino ($\propto 1/M^2$) but give an otherwise negligible correction to m_h, while the remaining operators induce a similar order effect for the LSP, via $\mathcal{O}(1/M^2)$ mixing with the MSSM terms.

Let us now discuss the CMSSM points with fine tuning $\Delta > 200$ i.e. situated above the black continuous line in Figure 2,3,4. They can bring an increase of m_h which can be significant, of 10-30 GeV (larger for $\mathcal{K}_0$), but this depends also on M. Therefore, points that in the MSSM would be eliminated by the LEP2 mass bound for Higgs $m_h > 114.4$ GeV, can now be ruled in as viable points. For example there are points which for m_h near 100 GeV can receive corrections of order 20 GeV or so, to now reach and satisfy the LEP2 bound. Interestingly, for $\mathcal{O}_{1,2}$ the Higgs mass increase is such that total m_h remains close to 120 GeV. In any case, only points that are largely fine tuned and have a value for m_h significantly below LEP2 bound, are actually receiving the largest corrections to m_h. Thus the phase space of the

MSSM is increased and more points which are otherwise ruled out on grounds of extreme fine tuning and/or LEP2 bound, are "recovered" and can be phenomenologically viable. The EW fine tuning Δ of those points can decrease by a factor equal to the square of the ratio of the Higgs masses after and before adding the correction δm_h, and this effect can be significant. For an example of how this works in the presence of the $d = 5$ operator $\mathcal{K}_0$, see [6], where one sees that Δ can remain acceptable (~ 10), in the presence of $\mathcal{K}_0$ even for m_h above 120 GeV. A similar effect is expected for the case of $d = 6$ operators.

6 Conclusions

In this paper we considered the extension of the MSSM Higgs sector by all possible effective operators of dimension-five and dimension-six, allowed by the MSSM symmetries. By supersymmetry, the same operators also provide the most general extension of the neutralino and chargino sectors of the MSSM. The study of such extensions is motivated by the attempts to understand better the MSSM higgs sector and its stability against corrections from new physics, as well as by dark matter studies. This is also motivated by the fact that dark matter and higgs sectors are intrinsically connected by supersymmetry. Complementary constraints from dark matter (large length scale physics) and electroweak physics (small length scales) can shed more light on either of these sectors or on both. In this paper we started an analysis in this direction, by computing for the first time, in component fields, the Lagrangian in the neutralino and chargino sectors extended by all effective operators of dimension $d = 5$ and $d = 6$, as well as the corresponding spectrum. The results can be used for studies of dark matter relic density within extensions of the CMSSM, by implementing this extended Lagrangian in public codes like micrOMEGAs. The study also continued our earlier similar calculation of the extended Lagrangian in the Higgs sector alone. The phenomenological impact of the effective operators was then studied by analyzing the impact of these operators on the CMSSM parameter space, as a perturbation.

We computed the mass corrections to the neutralino and chargino fields and showed that the neutralino LSP receives small mass corrections from individual effective operators, of few GeV ($< 1 - 2\%$) for a scale of the operators at 8 TeV; the sign of the corrections depends on the choice for the coefficients for these operators. The operators with the largest corrections were identified to be $\mathcal{O}_{2,6}$.

A similar study was done for the Higgs sector, and this continued the analysis started in our previous work [7], where the classical correction to the mass of the lightest MSSM higgs had been computed analytically, in the leading order ($1/M^2$). Using this analytical result we performed an accurate numerical investigation of the size of the correction to m_h from individual operators, including quantum corrections, not considered before. Using the CMSSM parameter space points that satisfied all electroweak and dark matter constraints,

except the LEP2 bound on m_h, we showed that points which would otherwise violate the LEP2 bound or are strongly fine tuned in CMSSM, become viable points, now respect this bound and have a lower fine-tuning. For such points, with $\Delta > 200$, the effective operators can bring *individual* corrections of $\delta m_h \sim 10 - 30$ GeV with the larger values for m_h further below LEP2 bound, to increase m_h just above this bound. Non-Susy effects associated with the effective operators can increase or decrease this correction. The properties of these phase space points need to be analyzed further in a global fit of the whole model i.e. MSSM plus effective operators.

An interesting result is that for the CMSSM phase space points with reduced EW fine tuning, $\Delta < 200$, and that satisfied the WMAP constraint within 3σ or were just consistent with this bound. In this case, the *supersymmetric* corrections to m_h from *individual* operators of dimension $d = 6$ were small: they were < 4 (< 6) GeV for $M = 10$ (8) TeV, respectively (with about a variation of 1 GeV for a change of 1 TeV of the scale M). The points below the LEP2 bound by this amount but respecting all other experimental and theoretical constraints, become now phenomenologically viable. The relative smallness of these corrections (for individual operators), suggests that the CMSSM "best fit" points are rather stable against the effects of "new physics" in the Higgs sector that could exist at 10 (8) TeV. In particular, for the CMSSM point with lowest EW fine tuning ($\Delta = 18$) that saturates the relic density within 3σ and predicting $m_h = 115.9 \pm 2$ GeV, and considering only corrections from individual operators, would bring this value to $m_h = (120 - 122) \pm 2$ GeV. This could suggest a preference for a light m_h even in the presence of "new physics" at $8 - 10$ TeV.

Acknowledgments

This work was supported in part by the European Commission under contracts PITN-GA-2009-237920 and ERC Advanced Grant 226371 ("MassTeV"), by the INTAS grant 03-51-6346, by the ANR (CNRS-USAR) contract 05-BLAN-007901 and by the CNRS PICS 3747 and 4172. The work of D.G. was supported in part by CNRS research contract 225933 held at Ecole Polytechnique Paris and ERC Advanced Grant 226371 ("MassTeV"). We thank Y. Mambrini (Orsay) for many interesting discussions, and S. Cassel (Oxford) for helping us with an updated set of phase space points of the CMSSM, at the 2-loop level, that was used in our numerical estimates.

7 Appendix

A The expressions of the $d = 6$ operators and the auxiliary fields.

The full component form of the dimension-six operators is

$$\begin{aligned}
\mathcal{O}_1 &= \frac{1}{M^2}\int d^4\theta\ \mathcal{Z}_1(S,S^\dagger)\ (H_1^\dagger\, e^{V_1}\, H_1)^2 \qquad \text{(A-1)}\\
&= 2\alpha_{10}\Big[|h_1|^2\left[\,|\mathcal{D}_\mu h_1|^2 + h_1^\dagger\,\frac{D_1}{2}\,h_1 + |F_1|^2\right] + |\,h_1^\dagger F_1|^2 + |h_1^\dagger \mathcal{D}^\mu h_1|^2\Big]\\
&+ 2\alpha_{10}\Big[\frac{i}{2}\,\overline{\psi}_1\overline{\sigma}^\mu\mathcal{D}_\mu\psi_1\,|h_1|^2 + \frac{i}{2}\,\overline{\psi}_1\overline{\sigma}^\mu\,\psi_1\,h_1^\dagger\mathcal{D}_\mu h_1 - \frac{i}{2}\,(h_1^\dagger\psi_1)\,\sigma^\mu\overline{\psi}_1(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_1 + h.c.\Big]\\
&+ 2\alpha_{10}\Big[-\frac{1}{\sqrt{2}}\,(h_1^\dagger\lambda_1\psi_1)|h_1|^2 - (h_1^\dagger\psi_1)(F_1^\dagger\psi_1) - \frac{1}{\sqrt{2}}\,(h_1^\dagger\psi_1)\,h_1^\dagger\lambda_1 h_1 + h.c.\Big]\\
&- \alpha_{10}(\overline{\psi}_1\psi_1)(\overline{\psi}_1\psi_1) + \Big[2\alpha_{11}m_0|h_1|^2\,(F_1^\dagger h_1) - \alpha_{11}m_0(\overline{\psi}_1 h_1)(\overline{\psi}_1 h_1) + h.c.\Big] + \alpha_{12}\,m_0^2(|h_1|^2)^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_2 &= \frac{1}{M^2}\int d^4\theta\ \mathcal{Z}_2(S,S^\dagger)\ (H_2^\dagger\, e^{V_2}\, H_2)^2 \qquad \text{(A-2)}\\
&= 2\alpha_{20}\Big[|h_2|^2\left[\,|\mathcal{D}_\mu h_2|^2 + h_2^\dagger\,\frac{D_2}{2}\,h_2 + |F_2|^2\,\right] + |h_2^\dagger F_2|^2 + |h_2^\dagger \mathcal{D}^\mu h_2|^2\Big]\\
&+ 2\alpha_{20}\Big[\frac{i}{2}\,\overline{\psi}_2\overline{\sigma}^\mu\mathcal{D}_\mu\psi_2\,|h_2|^2 + \frac{i}{2}\,\overline{\psi}_2\overline{\sigma}^\mu\,\psi_2\,h_2^\dagger\mathcal{D}_\mu h_2 - \frac{i}{2}\,(h_2^\dagger\psi_2)\,\sigma^\mu\overline{\psi}_2(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_2 + h.c.\Big]\\
&+ 2\alpha_{20}\Big[-\frac{1}{\sqrt{2}}\,(h_2^\dagger\lambda_2\psi_2)|h_2|^2 - (h_2^\dagger\psi_2)(F_2^\dagger\psi_2) - \frac{1}{\sqrt{2}}\,(h_2^\dagger\psi_2)\,h_2^\dagger\lambda_2 h_2 + h.c.\Big]\\
&- \alpha_{20}(\overline{\psi}_2\psi_2)(\overline{\psi}_2\psi_2) + \Big[2\alpha_{21}m_0|h_2|^2(F_2^\dagger h_2) - \alpha_{21}m_0(\overline{\psi}_2 h_2)(\overline{\psi}_2 h_2) + h.c.\Big] + \alpha_{22}m_0^2(|h_2|^2)^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_3 &= \frac{1}{M^2}\int d^4\theta\ \mathcal{Z}_3(S,S^\dagger)\ (H_1^\dagger\, e^{V_1}\, H_1)\,(H_2^\dagger\, e^{V_2}\, H_2), \qquad \text{(A-3)}\\
&= \alpha_{30}\Big[|h_1|^2\left[|\mathcal{D}_\mu h_2|^2 + h_2^\dagger\,\frac{D_2}{2}\,h_2 + |F_2|^2\right] + (h_1^\dagger F_1)(F_2^\dagger h_2) + (h_1^\dagger\mathcal{D}_\mu h_1)(h_2^\dagger\overleftarrow{\mathcal{D}}^\mu h_2) + (1\leftrightarrow 2)\Big]\\
&+ \alpha_{30}\Big[\frac{i}{2}\,\overline{\psi}_2\overline{\sigma}^\mu\mathcal{D}_\mu\psi_2\,|h_1|^2 + \frac{i}{2}\,\overline{\psi}_1\overline{\sigma}^\mu\,\psi_1\,h_2^\dagger\mathcal{D}_\mu h_2 - \frac{i}{2}\,(h_1^\dagger\psi_1)\,\sigma^\mu\overline{\psi}_2(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_2 + h.c.\Big]\\
&+ \alpha_{30}\Big[\frac{i}{2}\,\overline{\psi}_1\overline{\sigma}^\mu\mathcal{D}_\mu\psi_1\,|h_2|^2 + \frac{i}{2}\,\overline{\psi}_2\overline{\sigma}^\mu\,\psi_2\,h_1^\dagger\mathcal{D}_\mu h_1 - \frac{i}{2}\,(h_2^\dagger\psi_2)\,\sigma^\mu\overline{\psi}_1(\mathcal{D}_\mu - \overleftarrow{\mathcal{D}}_\mu)\,h_1 + h.c.\Big]\\
&+ \alpha_{30}\Big[-\frac{1}{\sqrt{2}}\,(h_2^\dagger\lambda_2\psi_2)|h_1|^2 - (h_1^\dagger\psi_1)(F_2^\dagger\psi_2) - \frac{1}{\sqrt{2}}\,(h_1^\dagger\psi_1)\,h_2^\dagger\lambda_2 h_2 + (1\leftrightarrow 2) + h.c.\Big]\\
&- \alpha_{30}\,(\overline{\psi}_1\psi_1)(\overline{\psi}_2\psi_2) + \Big[\alpha_{31}\,m_0\left[\,|h_1|^2(F_2^\dagger h_2) + |h_2|^2(F_1^\dagger h_1)\right] - \alpha_{31}^* m_0(h_1^\dagger\psi_1)(h_2^\dagger\psi_2) + h.c.\Big]\\
&+ \alpha_{32}\,m_0^2\,|h_1|^2|h_2|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_4 &= \frac{1}{M^2}\int d^4\theta\ \mathcal{Z}_4(S,S^\dagger)\ (H_2\,.\,H_1)\,(H_2\,.\,H_1)^\dagger, \qquad\qquad (\text{A-4})\\
&= \alpha_{40}\,|\partial_\mu(h_2.h_1)|^2+\alpha_{40}\Big[\frac{i}{2}\,(\psi_1.h_2+h_1.\psi_2)\,\sigma^\mu\partial_\mu(\psi_1.h_2+h_1.\psi_2)^\dagger+h.c.\Big]\\
&+ \alpha_{40}\,|h_2\cdot F_1+F_2\cdot h_1-\psi_2.\psi_1|^2+\Big[\alpha_{41}\,m_0\,(h_2.h_1)\,(h_2.F_1+F_2.h_1-\psi_2.\psi_1)^\dagger+h.c.\Big]\\
&+ \alpha_{42}\,m_0^2\,|h_2.h_1|^2
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_5 &= \frac{1}{M^2}\int d^4\theta\ \mathcal{Z}_5(S,S^\dagger)\ (H_1^\dagger\,e^{V_1}\,H_1)\,H_2.\,H_1+h.c. \qquad\qquad (\text{A-5})\\
&= \alpha_{50}\Big[\big[\,|\mathcal{D}_\mu h_1|^2+h_1^\dagger\,\frac{D_1}{2}\,h_1+|F_1|^2\,\big]\,(h_2.h_1)+(h_1^\dagger\overleftarrow{\mathcal{D}}_\mu h_1)\,\partial^\mu(h_2.h_1)\Big]\\
&+ \alpha_{50}^*\Big[i\,h_1^\dagger\,\mathcal{D}_\mu\psi_1\,\sigma^\mu\,(\psi_1.h_2+h_1.\psi_2)^\dagger+i\,(h_2.h_1)^\dagger\,\overline{\psi}_1\overline{\sigma}^\mu\mathcal{D}_\mu\psi_1\Big]\\
&+ \alpha_{50}\Big[(\psi_1.h_2+h_1.\psi_2)\,(F_1^\dagger\psi_1+\frac{1}{\sqrt{2}}\,h_1^\dagger\lambda_1 h_1)-\frac{1}{\sqrt{2}}(h_2.h_1)(h_1^\dagger\lambda_1\psi_1+\overline{\psi}_1\overline{\lambda}_1 h_1)\Big]\\
&+ \Big[\alpha_{50}(F_1^\dagger h_1)+\alpha_{51}^*\,m_0|h_1|^2\Big](h_2.F_1+F_2.h_1-\psi_2.\psi_1)-\alpha_{51}^*\,m_0\,(h_1^\dagger\psi_1)(h_2.\psi_1+\psi_2.h_1)\Big]\\
&+ m_0\,\Big[\alpha_{51}\,(F_1^\dagger h_1)+\alpha_{51}^*\,(h_1^\dagger F_1)\Big]\,(h_2.h_1)+\alpha_{52}\,m_0^2\,|h_1|^2\,(h_2.h_1)+\text{h.c. of all}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_6 &= \frac{1}{M^2}\int d^4\theta\ \mathcal{Z}_6(S,S^\dagger)\ (H_2^\dagger\,e^{V_2}\,H_2)\ H_2.\,H_1+h.c. \qquad\qquad (\text{A-6})\\
&= \alpha_{60}\Big[\big[\,|\mathcal{D}_\mu h_2|^2+h_2^\dagger\,\frac{D_2}{2}\,h_2+|F_2|^2\,\big]\,(h_2.h_1)+(h_2^\dagger\overleftarrow{\mathcal{D}}_\mu h_2)\,\partial^\mu(h_2.h_1)\Big]\\
&+ \alpha_{60}^*\Big[i\,h_2^\dagger\,\mathcal{D}_\mu\psi_2\,\sigma^\mu\,(\psi_1.h_2+h_1.\psi_2)^\dagger+i\,(h_2.h_1)^\dagger\,\overline{\psi}_2\overline{\sigma}^\mu\mathcal{D}_\mu\psi_2\Big]\\
&+ \alpha_{60}\Big[(\psi_1.h_2+h_1.\psi_2)\,(F_2^\dagger\psi_2+\frac{1}{\sqrt{2}}\,h_2^\dagger\lambda_2 h_2)-\frac{1}{\sqrt{2}}(h_2.h_1)(h_2^\dagger\lambda_2\psi_2+\overline{\psi}_2\overline{\lambda}_2 h_2)\Big]\\
&+ \Big[\alpha_{60}(F_2^\dagger h_2)+\alpha_{61}^*\,m_0|h_2|^2\Big](h_2.F_1+F_2.h_1-\psi_2.\psi_1)-\alpha_{61}^*\,m_0\,(h_2^\dagger\psi_2)(h_2.\psi_1+\psi_2.h_1)\Big]\\
&+ m_0\,\Big[\alpha_{61}\,(F_2^\dagger h_2)+\alpha_{61}^*\,(h_2^\dagger F_2)\Big]\,(h_2.h_1)+\alpha_{62}\,m_0^2\,|h_2|^2\,(h_2.h_1)+\text{h.c. of all}
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_7 &= \frac{1}{M^2}\sum_{s=w,y}\frac{1}{16g_s^2\kappa}\int d^2\theta\mathcal{Z}_7(S,0)\mathrm{Tr}(W^\alpha W_\alpha)_s(H_2.H_1)+h.c. \qquad\qquad (\text{A-7})\\
&= \sum_{s=w,y}\frac{\alpha_{70}^s}{4}\Big\{(h_2.h_1)\Big[i\,(\lambda_s^a\sigma^\mu\Delta_\mu\overline{\lambda}_s^a-\Delta_\mu\overline{\lambda}_s^a\overline{\sigma}^\mu\lambda_s^a)+D_s^aD_s^a-\frac{1}{2}(F_s^{a\,\mu\nu}F^a_{s\,\mu\nu}+\frac{i}{2}\epsilon^{\mu\nu\rho\sigma}F^a_{s\,\mu\nu}F^a_{s\,\rho\sigma})\Big]\\
&- \sqrt{2}\,(h_2.\psi_1+\psi_2.h_1)(\lambda_s^aD_s^a+\sigma^{\mu\nu}\lambda_s^aF^a_{s\,\mu\nu})+(h_2.F_1+F_2.h_1-\psi_2.\psi_1)\,\lambda_s^a\lambda_s^a\Big\}\\
&+ \frac{1}{4}\alpha_{71}^s m_0(h_2.h_1)(\lambda_s^a\lambda_s^a)+\text{h.c. of all,}\qquad (g_w\equiv g_2;\ \ g_y\equiv g_1;\ \ w:SU(2);\ \ y:U(1)).
\end{aligned}$$

$$\begin{aligned}
\mathcal{O}_8 &= \frac{1}{M^2}\int d^4\theta\ \Big[\mathcal{Z}_8(S,S^\dagger)\ \ [(H_2\,H_1)^2+h.c.]\Big] \qquad\qquad (\text{A-8})\\
&= 2\,\alpha_{81}^*\,m_0\,(h_2.h_1)\,(h_2.F_1+F_2.h_1-\psi_2.\psi_1)+m_0^2\,\alpha_{82}\,(h_2\cdot h_1)^2+\text{h.c. of all}
\end{aligned}$$

W^α is the Susy field strength of $SU(2)_L$ $(U(1)_Y)$ vector superfield V_w (V_y) of auxiliary component D_w (D_Y). Also

$$(1/M^2)\ \mathcal{Z}_i(S,S^\dagger) = \alpha_{i0} + \alpha_{i1}\, m_0\, \theta\theta + \alpha_{i1}^*\, m_0\, \overline{\theta\theta} + \alpha_{i2}\, m_0^2\, \theta\theta\overline{\theta\theta} \tag{A-9}$$

and $\mathcal{D}^\mu h_i = (\partial^\mu + i/2\, V_i^\mu)\, h_i$, $h_i^\dagger \overleftarrow{\mathcal{D}}^\mu = (\mathcal{D}^\mu h_i)^\dagger = h_i^\dagger(\overleftarrow{\partial}^\mu - i/2 V_i^\mu)$.

Further, $D_1 \equiv \vec{D}_w\, \vec{T} + (-1/2)\ D_Y$ and $D_2 \equiv \vec{D}_w\, \vec{T} + (1/2)\ D_Y$, $T^a = \sigma^a/2$. Finally, one must rescale in all $\mathcal{O}_i$ ($i \neq 7$ since $\mathcal{O}_7$ is rescaled already): $V_w \to 2\, g_2\, V_w$, $V_y \to 2\, g_1\, V_y$. Therefore one must replace $V_{1,2} = 2\, g_2\, \vec{V}_w\, \vec{T} + 2\, g_1\, (\mp 1/2)\, V_y$ with the upper sign (minus) for V_1, where $V_{1,2}$ enter above in the definition of $\mathcal{O}_{1,2}$. Similar expressions exist for the components of the superfields $V_{1,2}$. For example: $\lambda_{1,2} = g_2\, \lambda_w^a\, \sigma^a + g_1\, (\mp 1)\, \lambda_y$ (minus for λ_1).

Other notations used above: $H_1.H_2 = \epsilon^{ij}\, H_1^i\, H_2^j$. Also $|h_1 \cdot h_2|^2 = |h_1^i\, \epsilon^{ij}\, h_2^j|^2 = |h_1|^2\, |h_2|^2 - |h_1^\dagger\, h_2|^2$; $\epsilon^{ij}\, \epsilon^{kj} = \delta^{ik}$; $\epsilon^{ij}\, \epsilon^{kl} = \delta^{ik}\, \delta^{jl} - \delta^{il}\, \delta^{jk}$, $\epsilon^{12} = 1$, with

$$h_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix} \equiv \begin{pmatrix} h_1^1 \\ h_1^2 \end{pmatrix},\ Y_{h_1} = -1; \qquad h_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix} \equiv \begin{pmatrix} h_2^1 \\ h_2^2 \end{pmatrix},\ Y_{h_2} = +1 \tag{A-10}$$

In the above expressions for $\mathcal{O}_{1,2,\ldots 8}$, the notations h_i, ψ_i, $F_{1,2}$ stand for SU(2) doublets, so for example $h_1^\dagger \psi_1 = h_1^{i*}\, \psi_1^i$, also $|h_1|^2 = h_1^\dagger h_1 = h_1^{*i} h_1^i$, where the superscript i labels the SU(2) components, as shown above for the Higgs doublets $h_{1,2}$. Other notations: $\psi_1.h_2 = \psi_1^i \epsilon^{ij} h_2^j$ with a similar notation for the components of the doublet (superscripts). The derivatives $\mathcal{D}, (\overleftarrow{\mathcal{D}})$ only act on the first field to their right (left) respectively.

In $\mathcal{O}_7$ we used the notation:

$$\Delta_\mu \overline{\lambda}^a = \partial_\mu \overline{\lambda}^a - g\, t^{abc}\, V_\mu^b\, \overline{\lambda}^c, \qquad \Delta_\mu \overline{\lambda} = \partial_\mu \overline{\lambda} + \frac{i}{2}\, [V_\mu, \overline{\lambda}], \tag{A-11}$$

where the last equation applies before the rescaling of vector superfield (in matrix notation). These eqs are considered for λ_w (λ_y) with corresponding $V_{w,\mu}$ $(V_{y,\mu})$.

The Lagrangian with the above $\mathcal{O}_{1,..,8}, \mathcal{K}_0$ leads to (q is a doublet index)

$$\begin{aligned} F_1^{*q} &= -\big\{\epsilon^{qp}\, h_2^p\, \big[\mu + 2\, \zeta_{10}\, (h_1.h_2) + \rho_{11}\big] + h_1^{*q}\, \rho_{12} + \overline{\psi}_1^q\ \rho_{13}\big\} \\ F_2^{*q} &= -\big\{\epsilon^{pq}\, h_1^p\, \big[\mu + 2\, \zeta_{10}\, (h_1.h_2) + \rho_{21}\big] + h_2^{*q}\, \rho_{22} + \overline{\psi}_2^q\ \rho_{23}\big\} \end{aligned} \tag{A-12}$$

where ρ_{ij} are functions of $h_{1,2}$, given in

$$
\begin{aligned}
\rho_{11} &= -(2\alpha_{10}\,\mu+\alpha_{40}\mu+\alpha_{51}^*\,m_0)|h_1|^2-(\alpha_{30}\,\mu+\alpha_{40}\mu+\alpha_{61}^*\,m_0)\,|h_2|^2\\
&\quad -(\alpha_{41}^*\,m_0+\alpha_{50}^*\,\mu)\,(h_2.h_1)^*+\big[\,(\alpha_{60}+2\,\alpha_{50})\,\mu+2\alpha_{81}^*\,m_0\big]\,(h_1.h_2)\\
&\quad +\alpha_{40}\,\overline{\psi}_2.\overline{\psi}_1-(1/4)\ (\alpha_{70}^w\lambda_w^a\lambda_w^a+\alpha_{70}^y\lambda_y^2)\\
\rho_{12} &= \ (2\alpha_{11}^*\,m_0+\alpha_{50}^*\,\mu)|h_1|^2+(\alpha_{31}^*\,m_0+\alpha_{50}^*\,\mu)\,|h_2|^2\\
&\quad -\big[(2\alpha_{10}+\alpha_{30})\,\mu+\alpha_{51}^*\,m_0\big]\,(h_1.h_2)+\alpha_{51}^*\,m_0\,(h_2.h_1)^*-\alpha_{50}^*\,\overline{\psi}_2.\overline{\psi}_1\\
\rho_{13} &= -\big[2\alpha_{10}\,\overline{\psi}_1 h_1+\alpha_{30}\overline{\psi}_2 h_2-\alpha_{50}^*\,(\psi_1.h_2+h_1.\psi_2)^\dagger\big]
\end{aligned}
\tag{A-13}
$$

and

$$
\begin{aligned}
\rho_{21} &= -(2\alpha_{20}\,\mu+\alpha_{40}\mu+\alpha_{61}^*\,m_0)|h_2|^2-(\alpha_{30}\,\mu+\alpha_{40}\mu+\alpha_{51}^*\,m_0)\,|h_1|^2\\
&\quad -(\alpha_{41}^*\,m_0+\alpha_{60}^*\,\mu)\,(h_2.h_1)^*+\big[\,(\alpha_{50}+2\,\alpha_{60})\,\mu+2\alpha_{81}^*\,m_0\big]\,(h_1.h_2)\\
&\quad +\alpha_{40}\,\overline{\psi}_2.\overline{\psi}_1-(1/4)\ (\alpha_{70}^w\lambda_w^a\lambda_w^a+\alpha_{70}^y\lambda_y^2)\\
\rho_{22} &= \ (2\alpha_{21}^*\,m_0+\alpha_{60}^*\,\mu)|h_2|^2+(\alpha_{31}^*\,m_0+\alpha_{60}^*\,\mu)\,|h_1|^2\\
&\quad -\big[(2\alpha_{20}+\alpha_{30})\,\mu+\alpha_{61}^*\,m_0\big]\,(h_1.h_2)+\alpha_{61}^*\,m_0\,(h_2.h_1)^*-\alpha_{60}^*\overline{\psi}_2.\overline{\psi}_1\\
\rho_{23} &= -\big[2\alpha_{20}\,\overline{\psi}_2 h_2+\alpha_{30}\,\overline{\psi}_1\,h_1-\alpha_{60}^*\,(\psi_1.h_2+h_1.\psi_2)^\dagger\big]
\end{aligned}
\tag{A-14}
$$

The last line in ρ_{11}, ρ_{21} and ρ_{13}, ρ_{23} are new contributions, due to fermions only. Further

$$
\begin{aligned}
D_w^a &= -g_2\Big[\ h_1^\dagger T^a\,h_1\,(1+\tilde{\rho}_{1,w})+h_2^\dagger\,T^a\,h_2\,(1+\tilde{\rho}_{2,w})\ -\frac{\sqrt{2}}{4}\big(\alpha_{70}^w\,(h_2.\psi_1+\psi_2.h_1)\lambda_w^a+h.c.\big)\Big]\\
D_Y &= -g_1\Big[h_1^\dagger\frac{-1}{2}h_1(1+\tilde{\rho}_{1,y})+h_2^\dagger\frac{1}{2}h_2(1+\tilde{\rho}_{2,y})-\frac{\sqrt{2}}{4}\big(\alpha_{70}^y(h_2.\psi_1+\psi_2.h_1)\lambda_y^a+h.c.\big)\Big]
\end{aligned}
\tag{A-15}
$$

with $T^a=\sigma^a/2$, and

$$
\begin{aligned}
\tilde{\rho}_{1,w} &= 2\alpha_{10}|h_1|^2+\alpha_{30}|h_2|^2+\big[(\alpha_{50}-\alpha_{70}^w/2)\,(h_2.h_1)+h.c.\big]\\
\tilde{\rho}_{2,w} &= 2\alpha_{20}|h_2|^2+\alpha_{30}|h_1|^2+\big[(\alpha_{60}-\alpha_{70}^w/2)\,(h_2.h_1)+h.c.\big]
\end{aligned}
\tag{A-16}
$$

with similar expression for $\rho_{j,y}$ in which one uses instead α_{70}^y. Therefore

$$
\begin{aligned}
D_w^a\,D_w^a &= \frac{g_2^2}{4}\Big[\ \big(\ (1+\tilde{\rho}_{1,w})\,|h_1|^2-(1+\tilde{\rho}_{2,w})\,|h_2|^2\ \big)^2+4\,(1+\tilde{\rho}_{1,w})(1+\tilde{\rho}_{2,w})\,|h_1^\dagger\,h_2|^2\Big]\\
&\quad -\ \frac{\sqrt{2}}{2}\,g_2\Big[h_1^\dagger T^a h_1+h_2^\dagger T^a h_2\Big]\Big[\alpha_{70}^w\,(h_2.\psi_1+\psi_2.h_1)\,\lambda_w^a+h.c.\Big]\\
D_Y^2 &= \frac{g_1^2}{4}\big(\ (1+\tilde{\rho}_{1,y})\,|h_1|^2-(1+\tilde{\rho}_{2,y})\,|h_2|^2\,\big)^2\\
&\quad -\ \frac{\sqrt{2}}{2}g_1\Big[h_1^\dagger\frac{-1}{2}h_1+h_2^\dagger\frac{1}{2}\,h_2\Big]\Big[\alpha_{70}^y\,(h_2.\psi_1+\psi_2.h_1)\,\lambda_y+h.c.\Big]
\end{aligned}
\tag{A-17}
$$

B The neutralino and chargino Lagrangian.

Here we provide the full neutralino and chargino Lagrangian, in component fields (see notation (A-10) - similar notation for higgsino components), extracted from the total Lagrangian of Section 2. Below we use the notation:

$$\begin{aligned}
\partial^z_\mu f_1 &\equiv (\partial_\mu + (i/2)\, s\, g\, V^z_\mu) f_1 \qquad \overleftrightarrow{\partial}^z_\mu f_1 = (\partial_\mu - \overleftarrow{\partial}_\mu + i\, s\, g V^z_\mu)\, f_1 \\
\partial^\gamma_\mu f_2 &\equiv (\partial_\mu - (i/2)\, s\, g\, V^\gamma_\mu) f_2 \qquad \overleftrightarrow{\partial}^\gamma_\mu f_2 = (\partial_\mu - \overleftarrow{\partial}_\mu - i\, s\, g V^\gamma_\mu)\, f_2
\end{aligned} \tag{B-1}$$

where $s = +1$ for $f_1 = \psi_1^0,\, h_1^0,\, f_2 = \psi_1^-, h_1^-$ and $s = -1$ for $f_1 = \psi_2^0,\, h_2^0,\, f_2 = \psi_2^+, h_2^+$. Also

$$\begin{aligned}
V^\pm_\mu &= V^1_{w,\mu} \mp i\, V^2_{w,\mu} \equiv \sqrt{2}\, W^\pm_\mu, \qquad g\, V^z_\mu = g_2 V^3_{w,\mu} - g_1 V_{y,\mu}, \\
\lambda^\pm_w &= \lambda^1_w \mp i\, \lambda^2_w \equiv \sqrt{2}\, \tilde\lambda^\pm_w \qquad\qquad g\, V^\gamma_\mu = g_2 V^3_{w,\mu} + g_1\, V_{y,\mu}, \\
g &= g_2/\cos\theta_w \;=\; g_1/\sin\theta_w = e/(\sin\theta_w \cos\theta_w)
\end{aligned} \tag{B-2}$$

where $W^\pm_\mu$, $\tilde\lambda^\pm_w$ denote the charged weak bosons and charginos, respectively; (below we use the $V^\pm_\mu$ and $\tilde\lambda^\pm_w$ notation instead, to avoid complicating the expressions by the extra $\sqrt{2}$ factors).

The neutralino and chargino Lagrangian receives contributions from $\mathcal{L}$ of (5). $\mathcal{L}_D$ gives:

$$\begin{aligned}
\mathcal{L}_D \supset \frac{-1}{4\sqrt{2}} \Big[g_2 \alpha^w_{70} \Big(\lambda^3_w\, [|h_1^0|^2 - |h_2^0|^2 + |h_2^+|^2 - |h_1^-|^2] + \lambda^+_w\, (h_1^{0*} h_1^- + h_2^{+*} h_2^0) + \lambda^-_w (h_1^{0*} h_1^- + h_2^{+*} h_2^0)^* \Big) \\
- \; g_1\, \alpha^y_{70}\, \lambda_y \big[|h_1^0|^2 - |h_2^0|^2 + |h_1^-|^2 - |h_2^+|^2 \big] \Big] (\psi_1^0 h_2^0 + h_1^0 \psi_2^0 - \psi_1^- h_2^+ - h_1^- \psi_2^+) + h.c.
\end{aligned} \tag{B-3}$$

$\mathcal{L}_{F,1}$ contributes the neutralino/chargino terms below, given separately for each operator $\mathcal{O}_i$:

$$\mathcal{L}_{F,1} \;\supset\; \sum_i R_{\mathcal{O}_i} \qquad \text{where:} \tag{B-4}$$

$$\begin{aligned}
R_{\mathcal{O}_1} &= 2\,\alpha_{10}\, \mu\, (\psi_1^0\, h_2^0 - \psi_1^- h_2^+)\, (h_1^{0*} \psi_1^0 + h_1^{-*} \psi_1^-) + h.c. \\
R_{\mathcal{O}_2} &= 2\,\alpha_{20}\, \mu\, (\psi_2^0\, h_1^0 - \psi_2^+ h_1^-)\, (h_2^{0*} \psi_2^0 + h_2^{+*} \psi_2^+) + h.c. \\
R_{\mathcal{O}_3} &= \alpha_{30}\, \mu\, \big[(\psi_1^0\, h_2^0 - \psi_1^- h_2^+)\, (h_2^{0*} \psi_2^0 + h_2^{+*} \psi_2^+) \\
&\qquad + (\psi_2^0\, h_1^0 - \psi_2^+ h_1^-)\, (h_1^{0*} \psi_1^0 + h_1^{-*} \psi_1^-) \big] + h.c. \\
R_{\mathcal{O}_4} &= \alpha_{40}\, \mu\, (\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+)\, (|h_1^0|^2 + |h_2^0|^2 + |h_1^-|^2 + |h_2^+|^2) + h.c. \\
R_{\mathcal{O}_5} &= -\alpha_{50}\, \mu\, \big[(\psi_1^0 h_2^0 - \psi_1^- h_2^+)\, (\psi_1^0 h_2^0 + h_1^0 \psi_2^0 - \psi_1^- h_2^+ - h_1^- \psi_2^+) \\
&\qquad + (h_1^0 h_2^0 - h_1^- h_2^+)\, (\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+) \big] + h.c. \\
R_{\mathcal{O}_6} &= -\alpha_{60}\, \mu\, \big[(\psi_2^0 h_1^0 - \psi_2^+ h_1^-)\, (\psi_1^0 h_2^0 + h_1^0 \psi_2^0 - \psi_1^- h_2^+ - h_1^- \psi_2^+) \\
&\qquad + (h_1^0 h_2^0 - h_1^- h_2^+)\, (\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+) \big] + h.c. \\
R_{\mathcal{O}_7} &= \frac{1}{4}\, \mu\, \big[\alpha^w_{70}\, (\lambda^+_w \lambda^-_w + \lambda^3_w \lambda^3_w) + \alpha^y_{70}\, \lambda_y \lambda_y \big] \\
&\qquad \times (|h_1^0|^2 + |h_2^0|^2 + |h_1^-|^2 + |h_2^+|^2) + h.c.
\end{aligned} \tag{B-5}$$

Further, $\mathcal{L}_1$ generates the following terms, which are pairs of higgsino or of gaugino:

$$\mathcal{L}_1 = \sum_{i=1}^{8} S_{\mathcal{O}_i} \tag{B-6}$$

$$\begin{aligned}
S_{\mathcal{O}_1} &= i\,\alpha_{10}\Big[(|h_1^0|^2+|h_1^-|^2)\big[\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu^z\psi_1^0+(i/2)g_2(\overline{\psi_1^0}\overline{\sigma}^\mu V_\mu^+\psi_1^- + \overline{\psi_1^-}\overline{\sigma}^\mu V_\mu^-\psi_1^0)+\overline{\psi_1^-}\overline{\sigma}^\mu\partial_\mu^\gamma\psi_1^-\big] \\
&+ \quad (\overline{\psi_1^0}\overline{\sigma}^\mu\psi_1^0+\overline{\psi_1^-}\overline{\sigma}^\mu\psi_1^-)\big[h_1^{0*}\partial_\mu^z h_1^0+(i/2)g_2(h_1^{0*}V_\mu^+h_1^- + h_1^{-*}V_\mu^-h_1^0)+h_1^{-*}\partial_\mu^\gamma\,h_1^-\big]\Big] \\
&- (h_1^{0*}\psi_1^0+h_1^{-*}\psi_1^-)\sigma^\mu\big[\overline{\psi_1^0}\overleftrightarrow{\partial}{}_\mu^z\,h_1^0+ig_2(\overline{\psi_1^0}V_\mu^+h_1^-+\overline{\psi_1^-}V_\mu^-h_1^0)+\overline{\psi_1^-}\overleftrightarrow{\partial}{}_\mu^\gamma\,h_1^-\big]\Big]+h.c.
\end{aligned} \tag{B-7}$$

$$\begin{aligned}
S_{\mathcal{O}_2} &= i\,\alpha_{20}\Big[(|h_2^0|^2+|h_2^+|^2)\big[\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu^z\psi_2^0+(i/2)g_2(\overline{\psi_2^0}\overline{\sigma}^\mu V_\mu^-\psi_2^+ + \overline{\psi_2^+}\overline{\sigma}^\mu V_\mu^+\psi_2^0)+\overline{\psi_2^+}\overline{\sigma}^\mu\partial_\mu^\gamma\,\psi_2^+\big] \\
&+ \quad (\overline{\psi_2^0}\overline{\sigma}^\mu\psi_2^0+\overline{\psi_2^+}\overline{\sigma}^\mu\psi_2^+)\big[h_2^{0*}\partial_\mu^z h_2^0+(i/2)g_2(h_2^{0*}V_\mu^-h_2^+ + h_2^{+*}V_\mu^+h_2^0)+h_2^{+*}\partial_\mu^\gamma\,h_2^+\big]\Big] \\
&- \ (h_2^{0*}\psi_2^0+h_2^{+*}\psi_2^+)\sigma^\mu\big[\overline{\psi_2^0}\overleftrightarrow{\partial}{}_\mu^z\,h_2^0+ig_2(\overline{\psi_2^0}V_\mu^-h_2^++\overline{\psi_2^+}V_\mu^+h_2^0)+\overline{\psi_2^+}\overleftrightarrow{\partial}{}_\mu^\gamma\,h_2^+\big]\Big]+h.c.
\end{aligned} \tag{B-8}$$

$$\begin{aligned}
S_{\mathcal{O}_3} &= i\,\frac{\alpha_{30}}{2}\Big[(|h_1^0|^2+|h_1^-|^2)\big[\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu^z\psi_2^0+(i/2)g_2(\overline{\psi_2^0}\overline{\sigma}^\mu V_\mu^-\psi_2^++\overline{\psi_2^+}\overline{\sigma}^\mu V_\mu^+\psi_2^0)+\overline{\psi_2^+}\overline{\sigma}^\mu\partial_\mu^\gamma\,\psi_2^+\big] \\
&+ \quad (\overline{\psi_1^0}\overline{\sigma}^\mu\psi_1^0+\overline{\psi_1^-}\overline{\sigma}^\mu\psi_1^-)\big[h_2^{0*}\partial_\mu^z h_2^0+(i/2)g_2(h_2^{0*}V_\mu^-h_2^+ + h_2^{+*}V_\mu^+h_2^0)+h_2^{+*}\partial_\mu^\gamma\,h_2^+\big] \\
&- \quad (h_1^{0*}\psi_1^0+h_1^{-*}\psi_1^-)\sigma^\mu\big[\overline{\psi_2^0}\overleftrightarrow{\partial}{}_\mu^z\,h_2^0+ig_2(\overline{\psi_2^0}V_\mu^-h_2^++\overline{\psi_2^+}V_\mu^+h_2^0)+\overline{\psi_2^+}\overleftrightarrow{\partial}{}_\mu^\gamma\,h_2^+\big]\Big] \\
&+ \quad (|h_2^0|^2+|h_2^+|^2)\big[\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu^z\psi_1^0+(i/2)g_2(\overline{\psi_1^0}\overline{\sigma}^\mu V_\mu^+\psi_1^- + \overline{\psi_1^-}\overline{\sigma}^\mu V_\mu^-\psi_1^0)+\overline{\psi_1^-}\overline{\sigma}^\mu\partial_\mu^\gamma\,\psi_1^-\big] \\
&+ \quad (\overline{\psi_2^0}\overline{\sigma}^\mu\psi_2^0+\overline{\psi_2^+}\overline{\sigma}^\mu\psi_2^+)\big[h_1^{0*}\partial_\mu^z h_1^0+(i/2)g_2(h_1^{0*}V_\mu^+h_1^- + h_1^{-*}V_\mu^-h_1^0)+h_1^{-*}\partial_\mu^\gamma\,h_1^-\big]\Big] \\
&- \quad (h_2^{0*}\psi_2^0+h_2^{+*}\psi_2^+)\sigma^\mu\big[\overline{\psi_1^0}\overleftrightarrow{\partial}{}_\mu^z h_1^0+ig_2(\overline{\psi_1^0}V_\mu^+h_1^-+\overline{\psi_1^-}V_\mu^-h_1^0)+\overline{\psi_1^-}\overleftrightarrow{\partial}{}_\mu^\gamma h_1^-\big]\Big]+h.c.
\end{aligned} \tag{B-9}$$

$$S_{\mathcal{O}_4} = \frac{i\alpha_{40}}{2}\Big[(\psi_1^0h_2^0+h_1^0\psi_2^0-\psi_1^-h_2^+-h_1^-\psi_2^+)\sigma^\mu\partial_\mu(\psi_1^0h_2^0+h_1^0\psi_2^0-\psi_1^-h_2^+-h_1^-\psi_2^+)^\dagger+h.c.\Big]$$

$$\begin{aligned}
S_{\mathcal{O}_5} &= i\alpha_{50}^*\Big[\big[h_1^{0*}\partial_\mu^z\psi_1^0+(i/2)g_2(h_1^{0*}V_\mu^+\psi_1^-+h_1^{-*}V_\mu^-\psi_1^0)+h_1^{-*}\partial_\mu^\gamma\psi_1^-\big]\sigma^\mu(\psi_1^0h_2^0+h_1^0\psi_2^0 \\
&- \ \psi_1^-h_2^+-h_1^-\psi_2^+)^\dagger-(h_1^{0*}h_2^{0*}-h_1^{-*}h_2^{+*})\big[\overline{\psi_1^0}\overline{\sigma}^\mu\partial_\mu^z\psi_1^0+(i/2)g_2(\overline{\psi_1^0}\overline{\sigma}^\mu V_\mu^+\psi_1^-+\overline{\psi_1^-}\overline{\sigma}^\mu V_\mu^-\psi_1^0) \\
&+ \ \overline{\psi_1^-}\overline{\sigma}^\mu\partial_\mu^\gamma\psi_1^-\big]\Big]+h.c.
\end{aligned} \tag{B-10}$$

$$\begin{aligned}
S_{\mathcal{O}_6} &= i\alpha_{60}^*\Big[\big[h_2^{0*}\partial_\mu^z\psi_2^0+(i/2)g_2(h_2^{0*}V_\mu^-\psi_2^++h_2^{+*}V_\mu^+\psi_2^0)+h_2^{+*}\partial_\mu^\gamma\psi_2^+\big]\sigma^\mu(\psi_1^0h_2^0+h_1^0\psi_2^0 \\
&- \ \psi_1^-h_2^+-h_1^-\psi_2^+)^\dagger-(h_1^{0*}h_2^{0*}-h_1^{-*}h_2^{+*})\big[\overline{\psi_2^0}\overline{\sigma}^\mu\partial_\mu^z\psi_2^0+(i/2)g_2(\overline{\psi_2^0}\overline{\sigma}^\mu V_\mu^-\psi_2^++\overline{\psi_2^+}\overline{\sigma}^\mu V_\mu^+\psi_2^0) \\
&+ \ \overline{\psi_2^+}\overline{\sigma}^\mu\partial_\mu^\gamma\psi_2^+\big]\Big]+h.c.
\end{aligned} \tag{B-11}$$

$$S_{\mathcal{O}_7} = \frac{i}{2}\alpha_{70}^w\big[-h_1^0h_2^0+h_1^-h_2^+\big]\,\lambda_w^a\sigma^\mu\Delta_\mu\overline{\lambda_w^a}+\frac{i}{2}\alpha_{70}^y\,\big[-h_1^0h_2^0+h_1^-h_2^+\big]\,\lambda_y\sigma^\mu\partial_\mu\overline{\lambda_y}+h.c. \tag{B-12}$$

where

$$\begin{aligned}
\lambda_w^a \sigma^\mu \Delta_\mu \overline{\lambda_w^a} &= \frac{1}{2}\lambda_w^+ \sigma^\mu \partial_\mu \overline{\lambda_w^+} + \frac{1}{2}\lambda_w^- \sigma^\mu \partial_\mu \overline{\lambda_w^-} + \lambda_w^3 \sigma^\mu \partial_\mu \overline{\lambda_w^3} \\
&+ (i/2)\, g_2 \left[\lambda_w^+ \sigma^\mu (V_\mu^- \overline{\lambda_w^3} - V_w^3 \overline{\lambda_w^+}) + \lambda_w^- \sigma^\mu\, (V_w^3\, \overline{\lambda_w^-} - V_\mu^+ \overline{\lambda_w^3})\right. \\
&- \left.\lambda_w^3 \sigma^\mu \, (-V_\mu^+\, \overline{\lambda_w^+} + V_\mu^- \overline{\lambda_w^-})\right] \qquad \text{(B-13)}
\end{aligned}$$

Next, $\mathcal{L}_2$ of Section 2 gives contributions to the neutralino/chargino sectors:

$$\mathcal{L}_2 = \sum_{i=1}^{8} T_{\mathcal{O}_i} \qquad \text{(B-14)}$$

The Susy part of this contribution contains one gaugino and one higgsino (charged or not), while its non-Susy one contains two higgsinos (charged or not):

$$\begin{aligned}
T_{\mathcal{O}_1} &= \alpha_{10}\sqrt{2}\Big[-(|h_1^0|^2 + |h_1^-|^2)\big[g h_1^{0*}\lambda_z \psi_1^0 + g_2\,(h_1^{0*}\lambda_w^+\psi_1^- + h_1^{-*}\lambda_w^-\psi_1^0) - g h_1^{-*}\lambda_\gamma \psi_1^-\big] \\
&- (h_1^{0*}\psi_1^0 + h_1^{-*}\psi_1^-)\big[g\lambda_z |h_1^0|^2 + g_2\,(h_1^{0*}\lambda_w^+ h_1^- + h_1^{-*}\lambda_w^- h_1^0) - g|h_1^-|^2\lambda_\gamma\big]\Big] \\
&- \alpha_{11}^*\, m_0\, (h_1^{0*}\psi_1^0 + h_1^{-*}\psi_1^-)^2 + h.c. \qquad \text{(B-15)}
\end{aligned}$$

$$\begin{aligned}
T_{\mathcal{O}_2} &= \alpha_{20}\sqrt{2}\Big[-(|h_2^0|^2 + |h_2^+|^2)\big[-g h_2^{0*}\lambda_z \psi_2^0 + g_2\,(h_2^{0*}\lambda_w^-\psi_2^+ + h_2^{+*}\lambda_w^+\psi_2^0) + g h_2^{+*}\lambda_\gamma \psi_2^+\big] \\
&- (h_2^{0*}\psi_2^0 + h_2^{+*}\psi_2^+)\big[-g\lambda_z |h_2^0|^2 + g_2\,(h_2^{0*}\lambda_w^- h_2^+ + h_2^{+*}\lambda_w^+ h_2^0) + g|h_2^+|^2\lambda_\gamma\big]\Big] \\
&- \alpha_{21}^*\, m_0\, (h_2^{0*}\psi_2^0 + h_2^{+*}\psi_2^+)^2 + h.c. \qquad \text{(B-16)}
\end{aligned}$$

$$\begin{aligned}
T_{\mathcal{O}_3} &= \frac{\alpha_{30}}{\sqrt{2}}\Big[-(|h_1^0|^2 + |h_1^-|^2)\big[-g h_2^{0*}\lambda_z \psi_2^0 + g_2\,(h_2^{0*}\lambda_w^-\psi_2^+ + h_2^{+*}\lambda_w^+\psi_2^0) + g h_2^{+*}\lambda_\gamma \psi_2^+\big] \\
&- (h_1^{0*}\psi_1^0 + h_1^{-*}\psi_1^-)\big[-g\lambda_z |h_2^0|^2 + g_2\,(h_2^{0*}\lambda_w^- h_2^+ + h_2^{+*}\lambda_w^+ h_2^0) + g|h_2^+|^2\lambda_\gamma\big] \\
&- (|h_2^0|^2 + |h_2^+|^2)\big[g h_1^{0*}\lambda_z \psi_1^0 + g_2\,(h_1^{0*}\lambda_w^+\psi_1^- + h_1^{-*}\lambda_w^-\psi_1^0) - g h_1^{-*}\lambda_\gamma \psi_1^-\big] \\
&- (h_2^{0*}\psi_2^0 + h_2^{+*}\psi_2^+)\big[g\lambda_z |h_1^0|^2 + g_2\,(h_1^{0*}\lambda_w^+ h_1^- + h_1^{-*}\lambda_w^- h_1^0) - g|h_1^-|^2\lambda_\gamma\big] \\
&- \alpha_{31}^* m_0\ (h_1^{0*}\psi_1^0 + h_1^{-*}\psi_1^-)(h_2^{0*}\psi_2^0 + h_2^{+*}\psi_2^+) + h.c. \qquad \text{(B-17)}
\end{aligned}$$

$$T_{\mathcal{O}_4} = -\alpha_{41}^*\, m_0\, (h_1^{0*}\, h_2^{0*} - h_1^{-*} h_2^{+*})(\psi_1^0\ \psi_2^0 - \psi_1^-\ \psi_2^+) + h.c. \qquad \text{(B-18)}$$

$$\begin{aligned}
T_{\mathcal{O}_5} &= \frac{\alpha_{50}}{\sqrt{2}}\ \Big[(h_1^0\, h_2^0 - h_1^- h_2^+)\big[g h_1^{0*}\lambda_z \psi_1^0 + g_2\,(h_1^{0*}\lambda_w^+\psi_1^- + h_1^{-*}\lambda_w^-\psi_1^0) - g h_1^{-*}\lambda_\gamma \psi_1^- + h.c.\big] \\
&+ \big[g\,\lambda_z\,|h_1^0|^2 + g_2\,(h_1^{0*}\lambda_w^+ h_1^- + h_1^{-*}\lambda_w^- h_1^0) - g\lambda_\gamma\,|h_1^-|^2\big](\psi_1^0\, h_2^0 + h_1^0\psi_2^0 - \psi_1^-\, h_2^+ - h_1^-\, \psi_2^+)\Big] \\
&+ \alpha_{51}^*\, m_0 \Big[(h_1^{0*}\psi_1^0 + h_1^{-*}\, \psi_1^-)(\psi_1^0\, h_2^0 + h_1^0\psi_2^0 - \psi_1^-\, h_2^+ - h_1^-\, \psi_2^+) \\
&+ (|h_1^0|^2 + |h_1^-|^2)(\psi_1^0\psi_2^0 - \psi_1^-\psi_2^+)\Big] + h.c. \qquad \text{(B-19)}
\end{aligned}$$

$$
\begin{aligned}
T_{\mathcal{O}_6} &= \frac{\alpha_{60}}{\sqrt{2}} \Big[(h_1^0\, h_2^0 - h_1^-\, h_2^+)\big[-g h_2^{0*} \lambda_z \psi_2^0 + g_2\, (h_2^{0*} \lambda_w^- \psi_2^+ + h_2^{+*} \lambda_w^+ \psi_2^0) + g h_2^{+*} \lambda_\gamma \psi_2^+ + h.c\big] \\
&+ \big[-g\lambda_z |h_2^0|^2 + g_2 (h_2^{0*} \lambda_w^- h_2^+ + h_2^{+*} \lambda_w^+ h_2^0) + g\lambda_\gamma\, |h_2^+|^2\big]\, (\psi_1^0 h_2^0 + h_1^0 \psi_2^0 - \psi_1^- h_2^+ - h_1^- \psi_2^+)\Big] \\
&+ \quad \alpha_{61}^*\, m_0 \Big[(h_2^{0*} \psi_2^0 + h_2^{+*}\, \psi_2^+)(\psi_1^0\, h_2^0 + h_1^0 \psi_2^0 - \psi_1^-\, h_2^+ - h_1^-\, \psi_2^+) \\
&+ \quad (|h_2^0|^2 + |h_2^+|^2)(\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+)\Big] + h.c. \qquad \text{(B-20)}
\end{aligned}
$$

$$
T_{\mathcal{O}_7} = -\frac{1}{4} \alpha_{71}^w\, m_0\, (h_1^0\, h_2^0 - h_1^- h_2^+)(\lambda_w^3 \lambda_w^3 + \lambda_w^+ \lambda_w^-) - \frac{1}{4} \alpha_{71}^y\, m_0\, (h_1^0\, h_2^0 - h_1^- h_2^+)\, \lambda_y \lambda_y + h.c. \qquad \text{(B-21)}
$$

$$
T_{\mathcal{O}_8} = -2\alpha_{81}^*\, m_0 (h_1^0\, h_2^0 - h_1^- h_2^+)(\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+) + h.c. \qquad \text{(B-22)}
$$

$$
T_{\mathcal{K}_0} = -\zeta_{10} \big[2(h_1^0 h_2^0 - h_1^- h_2^+)(\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+) + (\psi_1^0\, h_2^0 + h_1^0 \psi_2^0 - \psi_1^-\, h_2^+ - h_1^-\, \psi_2^+)^2\big] + h.c. \qquad \text{(B-23)}
$$

$$
\begin{aligned}
g\lambda_z &\equiv g_2 \lambda_w^3 - g_1 \lambda_y \\
g\lambda_\gamma &\equiv g\lambda_w^3 + g_1 \lambda_y \qquad \text{(B-24)}
\end{aligned}
$$

Finally, $\mathcal{L}_3$ contains some four-chargino, four-neutralino as well as two-chargino-two-neutralino interaction terms:

$$
\begin{aligned}
\mathcal{L}_3 \supset &\; -\alpha_{10} (\overline{\psi_1^0} \psi_1^0 + \overline{\psi_1^-} \psi_1^-)^2 - \alpha_{20} (\overline{\psi_2^0} \psi_2^0 + \overline{\psi_2^+} \psi_2^+)^2 \\
&- \quad \alpha_{30} (\overline{\psi_1^0} \psi_1^0 + \overline{\psi_1^-} \psi_1^-)(\overline{\psi_2^0} \psi_2^0 + \overline{\psi_2^+} \psi_2^+) + \alpha_{40} (\overline{\psi_1^0 \psi_2^0} - \overline{\psi_1^- \psi_2^+})(\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+) \\
&+ \Big\{ 1/(2\sqrt{2}) \alpha_{70}^w \big[(\psi_1^0 h_2^0 - \psi_1^- h_2^+ + h_1^0 \psi_2^0 - h_1^- \psi_2^+) \sigma^{\mu\nu} \lambda_w^a F_{w,\mu\nu}^a + (\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+)(\lambda_w^3 \lambda_w^3 + \lambda_w^+ \lambda_w^-)\big] \\
&+ \; 1/(2\sqrt{2}) \alpha_{70}^y \big[(\psi_1^0 h_2^0 - \psi_1^- h_2^+ + h_1^0 \psi_2^0 - h_1^- \psi_2^+) \sigma^{\mu\nu} \lambda_y F_{\mu\nu} + (\psi_1^0 \psi_2^0 - \psi_1^- \psi_2^+)(\lambda_y \lambda_y)\big] \\
&+ \quad h.c. \Big\} \qquad \text{(B-25)}
\end{aligned}
$$

where a is an SU(2) index.

In applications not all operators are necessarily present. Depending on the cases considered, one can have only a subset of operators $\mathcal{O}_i, \mathcal{K}_0$ or just one of them, in which case the neutralino/chargino Lagrangian corrections of order $1/M^2$ simplify considerably. The Lagrangian in the neutralino/chargino sectors is then obtained, for each operator $\mathcal{O}_j$, by adding its contributions (identified by α_{jk}), from eqs.(B-3), (B-4), (B-6), (B-14), (B-25), plus the MSSM part.

C Chargino mass corrections from effective operators.

We use the notations:

$$\begin{aligned} m_w &= \frac{g_2 v}{2}; \quad \varphi = \big[m_2^2+\mu^2+2m_w^2\big]^2 - 4\big[m_2\mu + m_w^2\sin 2\beta\big]^2; \quad \alpha_{ij}^r = \frac{\alpha_{ij}+\alpha_{ij}^*}{2} \\ \tilde\lambda^\pm &\equiv \frac{1}{\sqrt2}(\lambda_w^1 \mp i\lambda_w^2); \qquad \psi_1 = \begin{pmatrix}\psi_1^0\\ \psi_-\end{pmatrix}, \quad \psi_2 = \begin{pmatrix}\psi_+\\ \psi_2^0\end{pmatrix}; \;\; \psi_1.\psi_2 = \psi_1^0\psi_2^0 - \psi_1^-\psi_2^+ \end{aligned} \quad \text{(C-1)}$$

The results for the chargino masses are:

$$m^2_{\tilde\chi^+_{1,2}} = m^2_{\tilde\chi^+_{1,2},\text{MSSM}} + \delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{K}_0) + \sum_i \delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_i) \quad \text{(C-2)}$$

where $i=1,..,8$ and $m^2_{\tilde\chi^+_{1,2},\,\text{MSSM}} = (1/2)\big[m_2^2+\mu^2+2m_w^2 \mp \sqrt{\varphi}\big]$ is the MSSM chargino mass with "$-$" for the lighter chargino $\tilde\chi_1^+$ and "$+$" for the heavier $\tilde\chi_2^+$. Also:

$$\begin{aligned}
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_1) &= \pm\frac{1}{2\sqrt\varphi}\alpha_{10}\, v^2\cos^2\beta\big[-m_2^2\mu^2+\mu^4 \mp \mu^2\sqrt\varphi + 2m_w^2\big(m_2^2+2\mu^2\mp\sqrt\varphi\big)\cos^2\beta \\
&+ 4m_w^4\cos^4\beta - 8m_2\mu m_w^2\cos\beta\sin\beta + 2\mu^2 m_w^2\sin^2\beta - m_w^4\sin^2 2\beta\big] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_2) &= \delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_1)[\alpha_{10}\to\alpha_{20}, \beta\to\pi/2-\beta] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_3) &= \mp\frac{\alpha_{30}\, v^2}{8\sqrt\varphi}\big[2m_2^2\mu^2 - 2\mu^4 - m_2^2 m_w^2 - 5\mu^2 m_w^2 \pm 2\mu^2\sqrt\varphi \pm m_w^2\sqrt\varphi \\
&+ m_w^2\big(m_2^2+\mu^2\mp\sqrt\varphi\big)\cos 4\beta + 8m_2\mu m_w^2\sin 2\beta\big] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_4) &= \pm\frac{\alpha_{41}^r m_0\, v^2\sin 2\beta}{4\sqrt\varphi}\big[\mu\big(m_2^2-\mu^2-2m_w^2\pm\sqrt\varphi\big)+2m_2 m_w^2\sin 2\beta\big] \\
&+ 2\delta m^2_{\mathcal{O}_3}[\alpha_{30}\to\alpha_{40}] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_5) &= \mp\frac{\alpha_{51}^r\; v^2\cos\beta}{2\sqrt\varphi}\big[m_0\mu\big(m_2^2-\mu^2-2m_w^2\pm\sqrt\varphi\big)\cos\beta + 4m_0 m_2 m_w^2\cos^2\beta\sin\beta\big] \\
&\mp \frac{\alpha_{50}^r\; v^2\cos\beta}{2\sqrt\varphi}\big[-2m_2\mu m_w^2\cos^3\beta + 2m_w^2\big(2m_2^2+4\mu^2-m_w^2\mp 2\sqrt\varphi\big)\cos^2\beta\sin\beta \\
&+ 4m_w^4\cos^4\beta\sin\beta + 2m_w^4\cos^2\beta\cos 2\beta\sin\beta - 2\mu\sin\beta\big[\mu\big(m_2^2-\mu^2\pm\sqrt\varphi\big) \\
&+ m_w^2\sin\beta(7m_2\cos\beta - 2\mu\sin\beta)\big]\big] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_6) &= \delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_5)[\alpha_{50}\to\alpha_{60}, \alpha_{51}\to\alpha_{61}, \beta\to\pi/2-\beta] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_7^w) &= \pm\frac{\alpha_{71w}^r\, v^2}{8\sqrt\varphi}\big[m_0\mu m_w^2 - m_0\mu m_w^2\cos 4\beta + m_0 m_2\big(-m_2^2+\mu^2-2m_w^2\pm\sqrt\varphi\big)\sin 2\beta\big] \\
&\pm\frac{\alpha_{70w}^r\, v^2}{8\sqrt\varphi}\big[2m_2\mu\big(m_2^2-\mu^2+4m_w^2\mp\sqrt\varphi\big) \pm 2\big(m_2^2+m_w^2\big)\sqrt\varphi\sin 2\beta - 2m_w^4\cos 4\beta\sin 2\beta \\
&- 4m_2\mu m_w^2\cos 4\beta - 2\big[m_2^2(m_2-\mu)(m_2+\mu) + 3\big(m_2^2+\mu^2\big)m_w^2 + m_w^4\big]\sin 2\beta\big]
\end{aligned} \quad \text{(C-3)}$$

$$
\begin{aligned}
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{O}_8) &= \pm\frac{\alpha^r_{81}}{2\sqrt{\varphi}} m_0 v^2 \sin 2\beta \big[\mu\big(m_2^2-\mu^2-2m_w^2\pm\sqrt{\varphi}\big)+2m_2 m_w^2\sin 2\beta\big] \\
\delta m^2_{\tilde\chi^+_{1,2}}(\mathcal{K}_0) &= \pm\frac{v^2\zeta_{10}\sin 2\beta}{2\sqrt{\varphi}}\big[\mu\big(m_2^2-\mu^2-2m_w^2\pm\sqrt{\varphi}\big)+2m_2 m_w^2\sin 2\beta\big] \\
&\pm \frac{v^4\zeta_{10}^2\sin^2 2\beta}{8\varphi^{3/2}}\big[2m_2^4\mu^2-4m_2^2\mu^4+2\mu^6-8m_2^2\mu^2 m_w^2+8\mu^4 m_w^2 \\
&+ 4m_2^2 m_w^4+8\mu^2 m_w^4+m_2^2\varphi-3\mu^2\varphi-2m_w^2\varphi\pm\varphi^{3/2}-4m_2^2 m_w^4\cos 4\beta \\
&+ 8m_2\,\mu\, m_w^2\big(m_2^2-\mu^2-2m_w^2\big)\sin 2\beta\big] \qquad\text{(C-4)}
\end{aligned}
$$

where the upper (lower) signs correspond to the lighter (heavier) chargino $\tilde\chi^+_1$ ($\tilde\chi^+_2$), respectively.

References

[1] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, "Observables In Low-Energy Superstring Models," Mod. Phys. Lett. A **1** (1986) 57.

[2] R. Barbieri and G. F. Giudice, "Upper Bounds On Supersymmetric Particle Masses," Nucl. Phys. B **306** (1988) 63;

[3] S. Cassel, D. M. Ghilencea and G. G. Ross, "Testing SUSY," Phys. Lett. B **687** (2010) 214 [arXiv:0911.1134 [hep-ph]]. "Testing SUSY at the LHC: Electroweak and Dark matter fine tuning at two-loop order," Nucl. Phys. B **835** (2010) 110 [arXiv:1001.3884 [hep-ph]].

[4] R. Barate *et al.* [LEP Working Group for Higgs boson searches], "Search for the standard model Higgs boson at LEP," Phys. Lett. B **565**, 61 (2003) [arXiv:hep-ex/0306033]; S. Schael *et al.* [ALEPH Collaboration], "Search for neutral MSSM Higgs bosons at LEP," Eur. Phys. J. C **47**, 547 (2006) [arXiv:hep-ex/0602042].

[5] G. Hinshaw *et al.* [WMAP Collaboration], "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations:Data Processing, Sky Maps, & Basic Results," Astrophys. J. Suppl. **180** (2009) 225 [arXiv:0803.0732 [astro-ph]].

[6] S. Cassel, D. M. Ghilencea and G. G. Ross, "Fine tuning as an indication of physics beyond the MSSM," Nucl. Phys. B **825** (2010) 203 [arXiv:0903.1115 [hep-ph]].

[7] I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, "MSSM Higgs with dimension-six operators," Nucl. Phys. B **831** (2010) 133 [arXiv:0910.1100 [hep-ph]].

[8] M. Carena, K. Kong, E. Ponton and J. Zurita, "Supersymmetric Higgs Bosons and Beyond," arXiv:0909.5434 [hep-ph].

[9] I. Antoniadis, E. Dudas, D. M. Ghilencea and P. Tziveloglou, "MSSM with Dimension-five Operators ($MSSM_5$)," Nucl. Phys. B **808** (2009) 155 [arXiv:0806.3778 [hep-ph]]. "Higher Dimensional Operators in the MSSM," AIP Conf. Proc. **1078** (2009) 175 [arXiv:0809.4598 [hep-ph]].

[10] I. Antoniadis, E. Dudas and D. M. Ghilencea, "Supersymmetric Models with Higher Dimensional Operators," JHEP **0803** (2008) 045 [arXiv:0708.0383 [hep-th]].

[11] D. Piriz and J. Wudka, "Effective operators in supersymmetry," Phys. Rev. D **56** (1997) 4170 [arXiv:hep-ph/9707314].

[12] A. Brignole, J. A. Casas, J. R. Espinosa and I. Navarro, "Low-scale supersymmetry breaking: Effective description, electroweak breaking and phenomenology," Nucl. Phys. B **666** (2003) 105 [arXiv:hep-ph/0301121].

[13] M. Dine, N. Seiberg and S. Thomas, "Higgs Physics as a Window Beyond the MSSM (BMSSM)," Phys. Rev. D **76** (2007) 095004 [arXiv:0707.0005 [hep-ph]].

[14] K. Blum, C. Delaunay and Y. Hochberg, "Vacuum (Meta)Stability Beyond the MSSM," arXiv:0905.1701 [hep-ph].

[15] M. Berg, J. Edsjo, P. Gondolo, E. Lundstrom and S. Sjors, "Neutralino Dark Matter in BMSSM Effective Theory," arXiv:0906.0583 [hep-ph].

[16] N. Bernal and A. Goudelis, "Dark matter detection in the BMSSM," JCAP **1003** (2010) 007 [arXiv:0912.3905 [hep-ph]].

[17] N. Bernal, "Neutralino dark matter detection beyond the MSSM," arXiv:1005.2116.

[18] K. Blum and Y. Nir, "Beyond MSSM Baryogenesis," Phys. Rev. D **78** (2008) 035005 [arXiv:0805.0097 [hep-ph]].

[19] N. Bernal, K. Blum, Y. Nir and M. Losada, "BMSSM Implications for Cosmology," JHEP **0908** (2009) 053 [arXiv:0906.4696 [hep-ph]].

[20] S. W. Ham, S. a. Shim and S. K. OH, "Possibility of spontaneous CP violation in Higgs physics beyond the minimal supersymmetric standard model," arXiv:0907.3300 [hep-ph]. "Explicit CP violation in the Dine-Seiberg-Thomas model," Eur. Phys. J. C **65** (2010) 517 [arXiv:0909.5098 [hep-ph]].

[21] P. Batra and E. Ponton, "Supersymmetric electroweak symmetry breaking," Phys. Rev. D **79** (2009) 035001 [arXiv:0809.3453 [hep-ph]].

[22] M. Carena, E. Ponton and J. Zurita, "BMSSM Higgs Bosons at the Tevatron and the LHC," Phys. Rev. D **82** (2010) 055025 [arXiv:1005.4887 [hep-ph]]. "SUSY Higgs bosons and beyond," PoS D **IS2010** (2010) 212 [arXiv:1006.5014 [hep-ph]].

[23] A. Brignole, F. R. Joaquim and A. Rossi, "Beyond the standard seesaw: neutrino masses from Kahler operators and broken supersymmetry," JHEP **1008** (2010) 133 [arXiv:1007.1942 [hep-ph]].

[24] G. Belanger, F. Boudjema, A. Pukhov and A. Semenov, "micrOMEGAs2.0: A program to calculate the relic density of dark matter in a generic model," Comput. Phys. Commun. **176**, 367 (2007) [arXiv:hep-ph/0607059].

[25] S. Groot Nibbelink and M. Hillenbach, "Renormalization of supersymmetric gauge theories on orbifolds: Brane gauge couplings and higher derivative operators," Phys. Lett. B **616** (2005) 125 [arXiv:hep-th/0503153]. S. G. Nibbelink and M. Hillenbach, "Quantum corrections to non-Abelian SUSY theories on orbifolds," Nucl. Phys. B **748** (2006) 60 [arXiv:hep-th/0602155]. D. M. Ghilencea, "Compact dimensions and their radiative mixing," Phys. Rev. D **70** (2004) 045018 [arXiv:hep-ph/0311264]. D. M. Ghilencea, H. M. Lee and K. Schmidt-Hoberg, "Higher derivatives and brane-localised kinetic terms in gauge theories on orbifolds," JHEP **0608** (2006) 009 [arXiv:hep-ph/0604215].

[26] D. M. Ghilencea and H. M. Lee, "Higher derivative operators from transmission of supersymmetry breaking on S(1)/Z(2)," JHEP **0509** (2005) 024 [arXiv:hep-ph/0505187]. "Higher derivative operators as loop counterterms in one-dimensional field theory orbifolds," JHEP **0503** (2005) 009 [arXiv:hep-ph/0409214].

[27] B. C. Allanach, "SOFTSUSY: a program for calculating supersymmetric spectra," Comput. Phys. Commun. **143** (2002) 305 [arXiv:hep-ph/0104145].

[28] M. M. El Kheishen, A. A. Aboshousha and A. A. Shafik, "Analytic Formulas For The Neutralino Masses And The Neutralino Mixing Matrix," Phys. Rev. D **45** (1992) 4345. A. Djouadi, J. Kalinowski, P. Ohmann and P. M. Zerwas, "Heavy SUSY Higgs bosons at e^+e^- linear colliders," Z. Phys. C **74** (1997) 93 [arXiv:hep-ph/9605339].